

REGRET THEORY: A BOLD ALTERNATIVE TO THE ALTERNATIVES*

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In their famous 1982 paper in this JOURNAL, Loomes and Sugden introduced regret theory. Now, more than 30 years later, the case for the historical importance of this contribution can be made.

Until the late 1970s, economists focused on the rational *homo economicus*, not only for normative but also for descriptive purposes. It was well understood that there were many empirical deviations from rationality, as signalled, for example, by preference reversals (Lichtenstein and Slovic, 1971; Lindman, 1971; Grether and Plott, 1979). But it was believed that irrational behaviour was too chaotic to be modelled and should just be taken as noise. For example, Arrow (1951, p. 406) wrote:

In view of the general tradition of economics, which tends to regard rational behavior as a first approximation to actual, I feel justified in lumping the two classes of theory [normative and descriptive] together.

The early 1980s saw a big shift, first in decision under risk, the topic of this article, and then in other fields including intertemporal choice, game theory and ambiguity (unknown probabilities). Kahneman and Tversky (1979) provided the first model of decision under risk that explicitly and deliberately deviated from the rational expected utility of *homo economicus*, but that could still be sufficiently tractable to permit economic modelling and predictions. Unfortunately, their model had some theoretical problems. It led their student Chew Soo Hong to co-author the unpublished Chew and MacCrimmon paper (1979), followed up by Chew (1983), with the first theoretically sound and axiomatised non-expected utility model. It also led John Quiggin (1982), then an unknown Australian student, to introduce his now famous rank-dependent utility. Machina (1982) gave a further boost to non-expected utility by providing constructive generalisations of optimality results. With the exception of Kahneman and Tversky, the aforementioned authors did not restrict their model to descriptive applications but also claimed a normative status of their models.

All the aforementioned generalisations maintained one of the most basic assumptions of economic optimisations: transitivity. Transitivity underlies the axioms of revealed preference for choices between multiple options. As good things often come

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This paper benefited from inputs from Mark Machina and comments from Graham Loomes and Robert F. Sugden.

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in threes, so it happened in 1982 when three papers independently proposed a theory that gave up transitivity: regret theory. One paper, Fishburn (1982), focused on mathematical and axiomatic elaborations. The second paper, Bell (1982), focused on decision analytic applications, taking regret as an extra attribute of consequences. The third paper, Loomes and Sugden (1982; LS henceforth), the topic of this review, focused on conceptual features and interpretations and most clearly described the empirical and normative status of regret theory. The three papers reinforced each other, with cross-references and mutual recognitions from the beginning.

Good ideas usually do not appear out of the blue but grow from seeds planted before. While the linguistic and the psychological concept of regret have existed for ages and have been studied in psychology for over a century (see Zeelenberg and Pieters, 2007 and their references), formal roles in decision theory have appeared since the 1950s. LS cite Savage's (1951) minimax regret theory and Fishburn (1988, p. 274) cites the bilinear mathematical functional of Kreweras (1961)¹ as a predecessor. Yet, it was not until 1982 that a complete decision theory of regret became available.

1. Regret Theory and Expected Utility

The LS paper, reproduced in this issue, gives a careful exposition of regret theory and its full details, with motivations and discussions added. The high quality and depth of their presentation has made the paper a classic. Our presentation aims to be didactical, focusing on the simplest and most popular special case of regret theory and on the simplest implications. Although our notation and terminology is usually as close as possible to LS, in a few instances we deviate and use conventions that are common in the field today.

$S = \{s_1, \dots, s_n\}$ denotes a *state space*, assumed finite for simplicity. Exactly one state s_j is true but a decision-maker is uncertain which state that is. Throughout this article, we use an example of an urn containing 100 balls numbered 1–100. One ball is drawn randomly. The true state of nature is the number of the ball actually drawn and $S = \{1, \dots, 100\}$, so that $n = 100$. Subsets of S are *events*, which are true if they contain the true state of nature. Thus, the event odd is $\{1, 3, \dots, 99\}$. *Actions*, with generic notation A , specify for each state s what the *consequence* $A(s)$ (money amount) is if s is true. In the example, a bet A on event odd, yielding £2 if s is odd and nothing otherwise but costing £1, would be the action A such that $A(s) = 1$ whenever s is odd and $A(s) = -1$ whenever s is even. We assume that S is endowed with a *probability measure* P , and write $p_j = P(s_j)$. In the example, every number has probability $1/100$, and every event with j states has probability $j/100$.

By \succcurlyeq we denote the preference relation of the decision-maker over actions, with strict preference \succ , indifference \sim and reversed preferences \preccurlyeq and \prec as usual. The most used model of decision under uncertainty is *expected utility* (EU). We then have

$$A_1 \succcurlyeq A_2 \Leftrightarrow \sum_{j=1}^n p_j C[A_1(s_j)] \geq \sum_{j=1}^n p_j C[A_2(s_j)] \quad (1)$$

¹ Fishburn learned about this work in French from personal communication with the French economist Denis Bouyssou.

for all actions A_1, A_2 . Here, C denotes the *utility function*, which is subjective. The probabilities may be objective, as in the example, but in the absence of objective information they are subjective. We can rewrite (1) as

$$A_1 \succcurlyeq A_2 \Leftrightarrow \sum_{j=1}^n p_j \{C[A_1(s_j)] - C[A_2(s_j)]\} \geq 0. \tag{2}$$

Table 1 illustrates a pair of actions, with M denoting *million*, A_R designating a risky action and A_S designating a safe action. In the Table, A_R yields 5M for ball numbers 1–10, 1M for numbers 11–99 and 0 for number 100.

Although A_R has the higher expected value, most people prefer A_S because of its safety, with no risk of ending up with 0. The regret of having missed a sure £1M if ball 100 is drawn is unbearable to many people. The preference for A_S can be accommodated by Bernoulli’s (1738) EU. Scaling $C(0) = 0$, substitution readily shows that the preference $A_S \succ A_R$ then holds if and only if

$$C(1M)/C(5M) > 10/11, \tag{3}$$

reflecting diminishing marginal utility.

We now consider a general choice situation between two actions A_1 and A_2 . Regret theory generalises expected utility by assuming that the utility $C[A_1(s_j)]$ experienced under A_1 is affected by what would have happened had A_2 been chosen instead of A_1 , and *vice versa*. People feel regret about $A_1(s_j)$ if the result of the alternative choice, $A_2(s_j)$, had been better. Because of this regret, under choice A_S in Table 1, people may feel less happy if ball 1–10 is drawn than if ball 11–99 is drawn, even though the same consequence, 1M, results in all these cases. If ball 1–10 is drawn, then winning £5M has been forgone due to an own decision, which arouses regret and reduces happiness relative to balls 11–99.

The other side of the coin of regret is rejoicing, felt if the most favourable consequence under some state s_j has resulted. After a choice A_S , people will rejoice if ball 100 is selected and, for the preference assumed in Table 2, this rejoicing is enough to prefer A_S , despite the regret felt for balls 1–10.

Regret theory holds if, for general actions A_1 and A_2 , we have

$$A_1 \succcurlyeq A_2 \Leftrightarrow \sum_{j=1}^n p_j Q \{C[A_1(s_j)] - C[A_2(s_j)]\} \geq 0. \tag{4}$$

The strictly increasing function Q captures the utility difference, but also the regret and rejoicing experienced at $A_1(s_j)$ and $A_2(s_j)$. Rejoicing being the other side of the regret-coin is captured by setting $Q(-x) = -Q(x)$. This equality ensures consistency of

Table 1
First Choice

Action	1	10	11	99	100
A_R		£5M		£1M	£0
A_S		£1M		£1M	£1M

Table 2
Second Choice

Action	1	10	11	99	100
A_r		£5M		£0	£0
A_s		£1M		£0	£1M

(4) when interchanging A_1 and A_2 . The equality implies the obvious $Q(0) = 0$. If Q is linear, then (4) does not offer any generalisation relative to EU (2), and utility C captures all that is relevant to decisions. New behavioural implications and a new decision theory result if Q is non-linear.

Besides (4), LS also consider more general representations

$$A_1 \succcurlyeq A_2 \Leftrightarrow \sum_{j=1}^n p_j Q[A_1(s_j), A_2(s_j)] \geq 0. \tag{5}$$

In (5), Q can depend on the pair of outcomes more generally than through their utility difference. We, however, focus on the tractable (4), the most popular special case used in the literature.

Before turning to the novelty of regret theory, we first discuss an important implication of EU that is preserved under regret theory: Savage’s (1954) *sure-thing principle* (see Table 2). Table 2 resulted from Table 1 by replacing the common outcome 1M, resulting under balls 11–99, by another common outcome, 0. Both EU and regret theory require that preference is not affected by such a change in common outcome. This condition, now known as Savage’s (1954) *sure-thing principle*, is implied by (4) as follows:

Proof. In (4), consider the substitution $A_1 = A_R$ and $A_2 = A_S$ (Table 1) and the alternative substitution $A_1 = A_r$ and $A_2 = A_s$ (Table 2). For both substitutions, the terms for $j = 11, \dots, 99$ cancel in the summation in (4) because they contribute 0 to the summation. After removing these 89 zero-terms, the summation in (4) is the same under both substitutions. Hence, we have the same inequality and the same preference for both substitutions, and thus for Tables 1 and 2.

The same implication and proof hold for EU, which is the special case where Q is the identity, and also for the more general (5). Under the following psychologically plausible scenario, the change in common outcome indeed does not affect choice. From Tables 1 and 2, subjects notice that their choice does not matter if ball 11–99 is drawn, because it leads to the same (common) outcome. They then decide to ignore these balls, after which they face the same (conditional or ‘isolated’) choice in the two cases. Focusing on balls 1–10 and 100, they perceive A_S as a pseudo-certain £1M (Kahneman and Tversky, 1979), making them prefer A_S as they preferred A_S in Table 1.

The displays and juxtapositions in Tables 1 and 2 enhance the aforementioned plausible scenario, and empirical studies have confirmed such isolation (Kahneman

and Tversky, 1979; see also LS p. 812).² Other scenarios, violating EU but not regret theory, can be triggered in other setups, which we discuss in the next Section.

2. Regret Theory’s Deviations from Expected Utility

2.1. A Deviation Illustrating the Regret Functional

This subsection discusses a theoretical deviation of regret theory from EU to illustrate the nature of the regret functional. The next subsections present empirical implications. Imagine that x_0, \dots, x_4 is an increasing sequence of outcomes that is equally spaced in C units. That is,

$$C(x_4) - C(x_3) = \dots = C(x_1) - C(x_0) > 0.^3 \tag{6}$$

We denote these utility differences by δ . Consider the two actions in Table 3.

Under expected utility, the two actions are equivalent because the C difference for balls 51–75 is twice as big as the C difference for balls 1–50 but it has half the probability. However, many decision-makers may prefer the lower action A_ℓ . They regret the small utility loss (x_3 instead of x_4) after choosing A_ℓ (balls 1–50) much less than the double and more salient utility loss (x_0 instead of x_2) after choosing A_u (balls 51–75). This is captured by $2Q[C(x_4) - C(x_3)] < Q[C(x_2) - C(x_0)]$. That is:

$$2Q(\delta) < Q(2\delta). \tag{7}$$

This condition is satisfied by functions Q that are convex on \mathbb{R}^+ (and, hence, concave on \mathbb{R}^-).

The reversed preference $A_u \succ A_\ell$ can also be accommodated by regret theory. Some decision-makers may prefer A_u because the probability of regret is only small (0.25 for balls 51–75), whereas the probability of regret is higher for A_ℓ (0.5 for balls 1–50). Such decision-makers do not discriminate much between utility losses δ and 2δ and for them the inequality in (7) is reversed. The most common case, however, is (7). It was recently confirmed empirically by Bleichrodt *et al.* (2010) and it is mostly assumed by LS (end of their Section II). Then extreme utility differences are salient and are overweighted. We now turn to some empirically important deviations from EU.

Table 3
Violation of Expected Utility Explained by Regret Theory

Action	1	25	26	50	51	75	76	100
A_u		x_4				x_0		0
A_ℓ		x_3		x_3		x_2		0

² Loomes and Sugden (1998), in yet another critical test of their theory, still found violations here, providing evidence against their theory. Birnbaum (2008, p. 481 ff.) also reports some violations.

³ Bleichrodt *et al.* (2010) demonstrated that these equalities can be revealed from preferences as follows. Using obvious notation, we measure indifferences (odd: x_{j+1} , even: g) \sim (odd: x_j , even: G) for $j = 0, \dots, 3$, and outcomes $G > g$ conveniently chosen. Equation (4) then implies $Q[C(G) - C(g)] = Q[C(x_j + 1) - C(x_j)]$ for all j . Because Q is strictly increasing, (6) follows.

Table 4
Violation of the Equivalence Axiom by Regret Theory

Action	1	25	26	50	51	75	76	100
A_d		£30		£20		£10		£0
A_n		£20		£10		£0		£30

2.2. Violating the Equivalence Axiom

See Table 4 (LS 6), where the subscript d in A_d refers to decreasing outcomes, whereas this is not the case for A_n .

Under the common assumption that large utility differences are overweighted, the superiority of A_n for balls 76–100 decides and A_n is preferred. Convexity of Q for gains implies this preference under regret theory:

$$0.25 \times Q[C(30) - C(20)] + 0.25 \times Q[C(20) - C(10)] + 0.25 \times Q[C(10) - C(0)] < 0.25 \times Q[C(30) - C(0)].$$

However, A_i and A_n induce the same probability distribution over outcomes! Apparently, such actions need not be equivalent under regret theory. Under EU, to the contrary, they must be equivalent, which LS (p. 818) calls the *equivalence axiom*. This requirement appears most clearly from (1). Then the correlation of the two actions, and the particular matching of their outcomes, is immaterial. It does not matter if A_n resulted from another independent drawing from the urn. By contrast, the matching of outcomes is crucial for regret theory, as is shown in (4). As surprising as this implication may be, it is a natural consequence if we experience regret. This point will be further discussed in the next subsection.

2.3. Accommodating the Allais Paradox, and a Comparison with Other Non-expected Utility Theories

Papers on non-expected utility of the 1980s usually started with a description of the Allais paradox and then showed how a newly introduced model could accommodate it. We now show how regret theory can accommodate this paradox. Consider a variation in Table 2, called the *independent variation*, where the lower action A_S is generated by a second, independent, drawing from the urn. Under EU's equivalence axiom, this change should not affect preference. However, under regret theory it may matter, because the matching of the outcomes changes and, hence, regret effects will change. We use a simple Q function to illustrate the basic idea. Imagine that the decision-maker feels no strong regret for utility losses up to $C(£0) - C(£1M)$ and $C(£1M) - C(£5M)$, and Q is close to linear for such and smaller losses. However, larger losses such as $C(£0) - C(£5M)$ exceed a tolerance threshold and result in strong regret. Choosing A_S risks experiencing such strong regret because, given the independence of the two actions and unlike the original choice situation in Table 2, outcome 5M for A_r and outcome 0 for A_S can occur simultaneously (with probability $0.10 \times 0.89 = 0.089$). If the regret $Q[C(£0) - C(£5M)]$ is strong enough, then A_r will be preferred.

The phenomenon just discussed is realistic. Experiments have shown that the exact presentation of actions matters, with Table 2 generating a pseudo-effect that disappears in the independent variation. Thus, there are more violations of the sure-thing principle in the independent variation (MacCrimmon, 1968; Moskowitz, 1974; Kahneman and Tversky, 1979; Starmer, 1992; Wu, 1994). The most detailed evidence is provided by Michael Birnbaum, whose branch independence concerns the test of the sure-thing principle controlling for regret in Table 2. A review of his work on this point is in Birnbaum (2008, p. 481 ff.).

Most recent experiments, aiming to investigate violations of the sure-thing principle, tested choices as in Tables 1 and 2 (with moderate pay-offs) but presented only the generated probability distributions to subjects, without specifying underlying states or joint distributions. Instead of the choice in Table 2, subjects then choose between two probability distributions (0.10: 5M, 0.90: 0) and (0.11: 1M, 0.89: 0), using an obvious notation. The majority preferences in Table 1 remain as indicated in that Table, with subjects still preferring certainty. But the preference in Table 2 is reversed, with the majority preference for A_2 , as in the aforementioned independent variation, and violating EU.⁴ LS put forward the plausible assumption that subjects take the probability distributions as independent if no joint distribution is specified. Then the analysis of our independent variation applies (LS Section III, 1st para.) and regret theory can accommodate the obtained violation of EU.

The violation of EU just discussed is known as the common consequence version of the Allais paradox (Allais, 1953). Allais' paradoxes spurred the non-expected utility models of the 1980s. These works, with regret theory as a prominent member, have led to what is called behavioural economics today. Most of the non-expected utility models abandon Savage's (1954) sure-thing principle jointly with its cousin under risk, von Neumann-Morgenstern's preferential independence. They allow for interactions between probabilities (beliefs) and utilities (tastes) that were excluded by expected utility. Although such interactions are interesting and can explain many phenomena, LS decided not to incorporate them in regret theory and instead explored another and bolder deviation. Regret theory allows for interactions between outcomes of different actions under the same state, which is excluded not only by expected utility but also by most other non-expected utility models. Dual interactions, between different outcomes of the same action under different states can similarly be considered. This is the topic of disappointment theory (Bell, 1985; Loomes and Sugden, 1987*a*; Delqu   and Cillo, 2006; Laciana and Weber, 2008). Although the distinction between regret and disappointment may sometimes be vague in natural language, decision theory strictly distinguishes between them.

A very general theory, accommodating virtually all empirical findings, results if we allow for all aforementioned interactions simultaneously. However, such a theory would be overly general, would become intractable and would not give useful implications or predictions. Hence, LS decided to include just one new interaction in regret theory, demonstrating that this already gives surprisingly many new and valuable

⁴ This follows because the equivalence axiom is violated. For an alternative derivation, the preferences just assumed imply $C(1M)/C(5M) < 10/11$ under EU, contradicting (3), so that EU cannot hold.

Table 5
A Preference Cycle Implied by Regret Theory

Action	1	25	26	50	51	75	76	100
A_i	0		10		20		30	
\wedge								
A_n	30		0		10		20	
\wedge								
A_a	20		30		0		10	
\wedge								
A_b	10		20		30		0	
\wedge								
A_i	0		10		20		30	

insights (see their Table 1) while maintaining tractability. Cubitt and Sugden (1998, p. 761) argue that giving up transitivity is like giving up separability but in another direction than when giving up the sure-thing principle. Although accommodating the Allais paradox was not LS's main goal, they could still do so.

2.4. *Violating Transitivity*

We now turn to the main goal of LS, incorporating the boldest and most controversial deviation from classical models available in the literature. Consider Table 5, an extension of Table 4 (combining LS's Tables 4 and 6), which is obtained by further shifts of outcomes. Each preference in the Table follows from the same line of reasoning as used in Table 4. The largest regret of 0 *versus* £30M each time overrules the multiple smaller regrets in the other direction. A preference cycle results and transitivity is violated. LS thus challenged one of the most standard assumptions of economic optimisations. They provided detailed arguments against transitivity (pp. 820–22), extended in later papers (Sugden, 1991, pp. 760–61).

Loomes and Sugden (1987*b*, beginning of Section 4) and Sugden (2004, Section II.7) showed that regret theory deviates from expected utility and can bring new phenomena only where it deviates from transitivity. Luce and Raiffa (1957, pp. 280–82) explained a similar point for earlier forms of regret. Hence, the violations of transitivity are central to regret theory. A generalisation is in Bikhchandani and Segal (2011, Theorem 1).

3. Empirical Support for Regret Theory

Regret theory received much support during the first decade after its introduction. Most empirical studies, several by Loomes and Sugden in collaboration with Chris Starmer, confirmed the predictions of the theory.

EXAMPLE 1. Loomes (1988*a*) tested the juxtaposition effects described in the preceding Section by asking subjects to state the money amount $\pounds a_0$ for which they were indifferent between the two actions in Table 6.

Table 6
First Choice

Action	1	40	41	100
A	£ a_0		£0	
B	£0		£12	

Table 7
Second Choice

Action	1	40	41	60	61	100
A'	£ a_1		£0		£0	
B'	£12		£12		£0	

Next in a second problem, subjects were asked to state the money amount £ a_1 for which they were indifferent between the two actions in Table 7.

Any theory based on the equivalence axiom predicts that $a_0 = a_1$. Regret theory makes a different prediction. The proof of the following claim is in the Appendix.

CLAIM 1. *Under regret theory with Q convex for gains, $a_1 > a_0$.*

Loomes (1988a) indeed found that the average value of a_1 was much larger than the average value of a_0 (£22.58 versus £17.52), confirming the prediction of regret theory.

Other studies on juxtaposition effects that supported regret theory include Loomes and Sugden (1987a), Loomes (1988b, 1989), Starmer and Sugden (1989) and Starmer (1992). Moreover, Loomes *et al.* (1992) confirmed violations of stochastic dominance predicted by regret theory.

A particularly desirable feature of regret theory is that it can explain preference reversals (PR). PRs were first discovered by Lichtenstein and Slovic (1971) and Lindman (1971), and were brought to the attention of economists by Grether and Plott (1979). PRs occur when subjects are confronted with two prospects, a £-bet which offers a relatively large sum of money, but a relatively small probability of winning, and a P-bet, which offers a more modest sum of money, but a greater probability of winning. Subjects are then asked to perform three tasks: to choose between the two prospects, and to attach a certainty equivalent to each prospect. The typical finding is that subjects prefer the P-bet, while paradoxically, the £-bet is given the higher valuation. The opposite pattern, choosing the £-bet but valuing the P-bet higher, is rarely observed.

Preference reversals challenge those who wish to explain economic behaviour in terms of rational theories of choice. Psychologists often interpreted PRs as evidence that individuals do not have a single system of preferences and respond differently to choice and valuation tasks (Slovic and Lichtenstein, 1983; Tversky *et al.*, 1988, 1990). Regret theory provides a different interpretation based on intransitive preferences as

Table 8
Third Choice

Action	1	30	31	60	61	100
£-bet		£18		£0		£0
P-bet		£8		£8		£0
C		£c		£c		£c

explained in the next example. Unlike any other existing theory, regret theory not only explains PRs but can even rationalise them.

EXAMPLE 2. Consider the three prospects in Table 8.⁵ The typical PR pattern is P-bet $>$ £-bet $>$ $c >$ P-bet for some sum of money c . This pattern can be explained by the extremity overweighting of Q in regret theory. For example, take $C(x) = x^{0.8}$ and $Q(x) = x^{1.5}$ for $x \geq 0$, and $C(-x) = -C(x)$ and $Q(-x) = -Q(x)$. Then for $c = 4$ regret theory accommodates the typical cycle, as calculations can show. The proof of the following claim is in the Appendix.

CLAIM 2. *Regret theory excludes opposite cycles.*

Loomes *et al.* (1989, 1992) and Loomes and Taylor (1992) found that the cycles predicted by regret theory were indeed much more common than the opposite cycles. They controlled for the psychological explanation that preference reversals are the result of differences in information processing between choice and valuation, and concluded that preference reversals were caused by intransitive preferences as predicted by regret theory.

If we reverse the signs of all sums of money in Table 8, turning gains into losses, then regret theory with concave Q for losses is consistent with the cycle £-bet $>$ P-bet $>$ $c >$ £-bet but not with the opposite cycle. Loomes and Taylor (1992) tested this prediction and concluded that their data again showed many more regret cycles than opposite cycles.

4. Challenges for Regret Theory

In the 1990s, some studies challenged the predictions of regret theory. Battalio *et al.* (1990) and Harless (1992) found that while regret effects occur using matrix presentations when states yielding the same consequence are collapsed, these regret effects become weaker for non-collapsed presentations. They also become weaker when problems are presented verbally or when other displays are used. Harless (1992, p. 647) suggested that regret effects are primarily framing effects that 'occur only when the decision is framed in a way that sharply directs the decision maker to compare acts and states'.

⁵ This is one of the problems considered in Loomes *et al.* (1992).

Table 9
Opposite Cycle

Action	1	10	11	15	16	20	21	100
A		£40		£0		£0		£0
B		£25		£25		£0		£0
C		£15		£15		£15		£0

Another challenge to regret theory came from other studies, starting with Tversky (1969), that observed systematic cycles that could not be explained by regret theory. Consider the three actions in Table 9, which is problem T'U'V' in Day and Loomes (2010). Regret theory is consistent with the cycle $C \succ B \succ A \succ C$ but not with the opposite cycle. However, Day and Loomes (2010) observed that this opposite cycle prevailed. Their finding can be explained by Rubinstein's (1988) similarity theory. When comparing A and B , people may consider that the 5% extra probability that B offers is so small that they pay little or no attention to the probability dimension and instead concentrate on the dissimilar pay-off dimension and choose A . Likewise, they consider the winning probabilities of B and C to be similar and choose B . However, they may also find that the 10% difference in winning probability between A and C is large enough to make A and C look dissimilar on the probability dimension and this may shift their attention back to the probability dimension and they then choose C . Lindman and Lyons (1978), Budescu and Weiss (1987), Leland (1994, 1998), Mellers and Biagini (1994), Bateman *et al.* (2007) and Day and Loomes (2010) reported evidence for such similarity cycles, which cannot be explained by regret theory. Starmer (1999) reported a comparable cycle although he did not explain it by similarity but by original prospect theory (Kahneman and Tversky, 1979).

Yet another challenge came from mathematical psychologists. Starting with Iverson and Falmagne (1985), several papers showed that asymmetric cycles need not necessarily be inconsistent with transitive preferences if the stochastic nature of human preferences is taken into account (overviewed by Regenwetter *et al.*, 2011). Even though they mainly concentrated on the similarity cycles observed by Tversky (1969) and showed that these could be explained by transitive preferences with error, their objections also applied to the regret cycles that were observed.

However, a serious blow to regret theory came from Starmer and Sugden (1993). They discovered that previously observed support for regret theory could, to a large extent, be explained by event-splitting effects by which splitting an event with a given consequence into two sub-events increases its weight.

EXAMPLE 3. Consider the four problems in Tables 10–13. According to regret theory, Problems I and III are equivalent and so are Problems II and IV. Regret theory predicts that choices AB' (A in Problems I and III and B' in Problems II and IV) will occur more often than choices BA' . However, according to event splitting, choices AB' should be more likely than choices BA' in Problems I and II, suggesting regret effects,

Table 10
Problem I

Action	1	45	46	100
A		£11		£0
B		£0		£7

Table 11
Problem II

Action	1	45	46	55	56	100
A'		£11		£11		£0
B'		£7		£7		£0

Table 12
Problem III

Action	1	45	46	90	91	100
A		£0		£11		£0
B		£7		£0		£7

Table 13
Problem IV

Action	1	45	46	90	91	100
A'		£11		£0		£11
B'		£7		£0		£7

but not in Problems III and IV: in Problem I the £7 is not split, whereas in Problem II it is, which may make B' appear more attractive, but both in Problems III and in Problem IV the £7 is split.

The prediction of event splitting was, indeed, what Starmer and Sugden (1993) observed: clear regret effects in Problems I and II but no effects in Problems III and IV. Their study suggested strong event-splitting effects and weaker regret effects (see also Humphrey, 1995). On the other hand, Starmer and Sugden (1998) found that not all regret effects were due to event-splitting effects but that some were mainly due to framing, as had been suggested before by Harless (1992).

Starmer and Sugden subjected 'their' regret theory to rigorous testing⁶ and thereby discovered the remarkable fact that splitting states can make prospects substantially more attractive. Camerer (1995, pp. 655–56) praised the authors' work on regret theory, and the resulting progress of our understanding, and wrote 'this is a story of successful detective work'.

5. Recent Applications

Even though event-splitting effects may provide an alternative explanation for some of the phenomena that led to the introduction of regret theory, as Starmer (2000, p. 376) notes 'insights from [regret theory] have proved useful in understanding real behaviour'. The authors of this article benefited from regret theory's insight that pairs of outcomes for different actions provide a natural basis for decision-making and used this idea in trade-off techniques (Bleichrodt *et al.*, 2010; Wakker, 2010). This insight was also used by Bouyssou and Pirlot (2003, especially table 1) and Vind (2003). The 2000s have witnessed many applications that either use regret theory or extensions of the model, with LS cited as a source of inspiration. For example, Barberis *et al.* (2006) use regret theory to explain the stock market participation puzzle: few people invest in stocks even though rational economic theory predicts that they should. Other applications of regret models to financial decisions include Muermann *et al.* (2006) and Michenaud and Solnik (2008) who study asset allocation decisions.

Braun and Muermann (2004) apply regret theory to the demand for insurance and show that regret theory can explain the frequently observed preference for low deductibles. Smith (1996) applies regret theory to health and Perakis and Roels (2008) use it in the newsvendor model. Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2008) explain how regret theory can explain overbidding in first price auctions. Other regret models include Sarver (2008) and Hayashi (2008). These models differ from LS in that they study preferences over menus, i.e. sets of prospects, in which decision-makers experience regret if their choice turns out to be inferior *ex post*.

A critical aspect of regret is the extent to which decision-makers, after their choices, are informed about the outcomes that would have resulted had they chosen differently. This issue has been explored in the experimental and theoretical literature on feedback-conditional regret. It has been found that people prefer options which screen them from discovering the outcome of forgone choices. The anticipated pain of regret is reduced or eliminated if people do not know the outcome of the forgone choice. Thus, the option of not entering a lottery is more attractive if, conditional on not entering, one will never know whether one would have won or lost. This tendency is exploited in postal code lotteries in which a postal code rather than an anonymous number is drawn (Zeelenberg, 1999; Humphrey, 2004).

Regret theory has been widely applied in the health domain, raising fundamental ethical questions. Should doctors be allowed to use excessive diagnostic testing just to avoid the regret about missing the occasional serious case, just because they

⁶ As was done in many papers by Loomes and Sugden, including Loomes and Sugden (1998).

overweight omission relative to commission (Ritov and Baron, 1995)? Should one-sided legal liability be imposed on doctors to induce such regret and overweighting externally, at the cost of societal efficiency? Should tests for Down syndrome and vaccinations that demonstrably reduce the mortality rate be provided to the general public even though they may lead to lifelong emotions of regret that would not have occurred otherwise (Ritov and Baron, 1990; Murray and Beattie, 2001)? Or in another domain, should seeding hurricanes be forbidden if it leads to regret with some parties affected, even though total damage is reduced (Howard *et al.*, 1972)?

The 2000s has seen the emergence of neuroeconomics which has led to new insights into regret. Camille *et al.* (2004) find that the orbitofrontal cortex has a fundamental role in mediating regret and that people with lesions in the orbitofrontal cortex⁷ who do not experience regret make worse decisions than normal subjects who do anticipate regret. Giorgetta *et al.* (2013) found different neural localisations for regret and disappointment.

Bleichrodt *et al.* (2010) developed methods to obtain precise quantitative measurements of the parameters of regret theory. These measurements allow us to derive exact predictions, for example, about how much more supply is needed next year if regret is increased by advertisement campaigns. To illustrate another application, in Example 2 we showed that there are values of C and Q for which regret theory predicts preference reversals. By measuring these values individually, we can predict exactly when preference reversals will occur for each subject and we can then test whether they actually do (Baillon *et al.*, 2014).

Whereas regret theory accommodates intransitivities by allowing state-wise comparisons of consequences, it maintains the classical linear weighting of probabilities. Two recent approaches relax the latter assumption. Loomes's (2010) new model, the perceived relative argument model, is a rich model defined for the probability triangle and uses paired comparisons of consequences like regret theory, but it also uses similar paired comparisons of probabilities. It can explain many empirical regularities including the aforementioned similarity cycles that are inconsistent with regret theory.

Bordalo *et al.*'s (2012) salience theory uses pairwise comparisons of consequences to readjust the weights (salience), rather than utility differences, of states of nature. As did LS, salience theory assumes that large differences are overweighted but it does not use an extremity overweighting function Q for differences of utilities to model this. Instead, it assumes that the salience function overweights the states of nature that have large utility differences for the actions under consideration. Salience theory shares the implication of the sure-thing principle with LS, with the salience of state s not affected by consequences outside s . As with regret theory, the novelty of salience theory resides in where it violates transitivity. Unlike LS, Bordalo *et al.* (2012) did not analyse or discuss intransitivities extensively, but left this to future work.

⁷ People with lesions in the orbitofrontal cortex are not emotionally unresponsive as they did experience disappointment.

6. Discussion

LS were not only bold in taking issue with some of the most widely accepted assumptions in decision theory, transitivity and the equivalence axiom but also in their interpretations, showing insights ahead of their time. When their paper was published in the early 1980s, a strict ordinal revealed preference view was dominant in economics. Utility modelled decisions and nothing else. Introspective interpretations were not made. The situation has changed today, with Kahneman (1994), Loewenstein and Ubel (2008, Section 2) and others pleading for broader interpretations of utility, and with introspective happiness studies, a popular and influential field in economics (van Praag and Ferrer-i-Carbonell, 2004; Diener and Biswas-Diener, 2008; Benjamin *et al.*, 2014). Although LS could have avoided introspection because, as they show in their Appendix, all components in their model can be revealed from preferences, they chose a psychologically sound interpretation of their model: They interpreted the function C in Section 3 as the inherent utility, resulting when the individual experiences a consequence ‘*without having chosen it*’ (emphasis in original). Then regret or rejoicing plays no role. C can be felt through introspection. Hence, LS used the term choiceless utility for C . Next, in a second stage, regret comes in, captured through the function Q . LS (Section V, 2nd para.) explicitly distanced themselves from a narrow empirical approach to preference theory.

In 1982, the prevailing hypothesis of prospect theory was a total reflection of preference, with risk aversion for gains coupled with equally strong risk seeking for losses. LS (Section III end) immediately predicted weaker, only partial, reflection with risk seeking for losses weaker than risk aversion for gains. Their prediction has since been confirmed empirically (surveyed by Wakker, 2010 Section 9.5). LS also carefully presented evidence against regret theory (Section V, middle) and recommend reference-dependent generalisations.

A limitation of regret theory, as of any intransitive theory of binary choice, is that it is unclear how to extend the theory to choices among three or more actions. LS (Section IV) provided the first ideas about such extensions, with defences against book making and money pump criticisms in Section V; Loomes and Sugden (1987*b*) provided an elaborated theory. A preference foundation is in Sugden (1993). Hayashi (2008) suggested an alternative extension.

Although LS are firm on a normative status of regret theory and provide strong and cogent arguments, the authors of this comment have different views. LS argue that feelings of regret are a fact of life and that it is irrational to ignore them, a view supported by Bourgeois-Gironde (2010) using neurodata. We are less tolerant and more paternalistic about such feelings. In its everyday meaning, regret is a useful emotion to signal possible improvements of future actions in situations of incomplete information. The formal decision-theoretic meaning, however, is different. Consider Table 2, with a choice of A . A rational person should maximise happiness, given the external constraints. The latter are the same if ball 11 is drawn as if ball 100 is drawn, in both cases the consequence being £0. Having feelings of regret for ball 100 because of the forgone £1M leads to harm for no good reason. We believe that such voluntary self-harming is irrational.

Note that, unlike in everyday life situations where regret can be a useful signal, nothing can be learned from the ball drawn in Table 7, given that all probabilities and consequences were known beforehand. We also assume complete modelling and, hence, for instance, we assume that there are no outsiders blaming the decision-maker after ball 100 was drawn. Taking any emotion as rational just because it exists is too permissive and applies Hume's adage 'reason is, and ought only to be the slave of the passions' too leniently. Although we see no normative status for regret theory, it is obvious that its descriptive value is huge, making it one of the most important contributions to decision theory. LS's careful arguments for the rationality of regret theory, challenging something as basic as transitivity, are thought provoking and have also produced many new insights.

7. Conclusion

In our perception, salient features of Sugden's work during the last three decades have been great originality and breadth, and salient features of Loomes' work have been great sharpness and depth. In retrospect, it is then no surprise that when these two strong and complementary minds came together in 1982, something lasting resulted.

Appendix A. Proofs

Proof of claim 1. Informally, writing a for both a_0 and a_1 , in Table 7, 40 differences (£ a versus £0) and 40 differences (£0 versus £12) of Table 6 have been replaced by 40 differences (£ a versus £12) and 40 differences (£0 versus £0). By the extremity overweighting of Q , the removals of (£ a versus £0) count most, weakening the case for the upper prospect. Hence, a larger value a_1 is needed in Table 7.

Formally, according to regret theory the first indifference implies

$$0.40 \times Q[C(a_0) - C(0)] = 0.60 \times Q[C(12) - C(0)]. \quad (\text{A.1})$$

The second indifference implies

$$0.40 \times Q[C(a_1) - C(12)] = 0.20 \times Q[C(12) - C(0)]. \quad (\text{A.2})$$

By Q 's extremity overweighting, $Q[C(a_0) - C(0)] > Q[C(a_0) - C(12)] + Q[C(12) - C(0)]$. Hence $0.40 \times Q[C(a_0) - C(12)] < 0.40 \times Q[C(a_0) - C(0)] - 0.40 \times Q[C(12) - C(0)] =$ (by (A.1)) $0.60 \times Q[C(12) - C(0)] - 0.40 \times Q[C(12) - C(0)] = 0.20 \times Q[C(12) - C(0)]$. Because Q is strictly increasing, it follows that to obtain the equality in (A.2) we must have $a_1 > a_0$.

Proof of claim 2. For contradiction, assume the opposite cycle P-bet $<$ £-bet $<$ c $<$ P-bet. Then

$$0.30Q[C(18) - C(8)] + 0.30Q[-C(8)] > 0, \quad (\text{A.3})$$

$$0.30Q[C(8) - C(c)] + 0.30Q[C(8) - C(c)] + 0.40Q[-C(c)] > 0, \quad (\text{A.4})$$

$$0.30Q[C(c) - C(18)] + 0.30Q[C(c)] + 0.40Q[C(c)] > 0. \quad (\text{A.5})$$

Adding the left-hand sides of (A.3)–(A.5) and using $Q(x) = -Q(-x)$ for all $x > 0$ gives

$$\begin{aligned} &0.30\{Q[C(18) - C(8)] + Q[C(8) - C(c)] - Q[C(18) - C(c)]\} + \\ &0.30\{-Q[C(8)] + Q[C(8) - C(c)] + Q[C(c)]\} > 0. \end{aligned} \quad (\text{A.6})$$

Because Q overweights extremes, the terms in square brackets are negative and we have a contradiction.

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Submitted: 18 March 2014

Accepted: 18 June 2014

References

- Allais, M. (1953). 'Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine', *Econometrica*, vol. 21(4), pp. 503–46.
- Arrow, K.J. (1951). 'Alternative approaches to the theory of choice in risk-taking situations', *Econometrica*, vol. 19(4), pp. 404–37.
- Baillon, A., Bleichrodt, H. and Cillo, A. (2014). 'A tailor-made test of intransitive choice', *Operations Research*, (forthcoming).
- Barberis, N., Huang, M. and Thaler, R.H. (2006). 'Individual preferences, monetary gambles, and stock market participation: a case for narrow framing', *American Economic Review*, vol. 96(4), pp. 1069–90.
- Bateman, I.J., Day, B., Loomes, G. and Sugden, R.F. (2007). 'Can ranking techniques elicit robust values?', *Journal of Risk and Uncertainty*, vol. 34(1), pp. 49–66.
- Battalio, R.C., Kagel, J.H. and Jiranyakul, K. (1990). 'Testing between alternative models of choice under uncertainty: some initial results', *Journal of Risk and Uncertainty*, vol. 3(1), pp. 25–50.
- Bell, D.E. (1982). 'Regret in decision making under uncertainty', *Operations Research*, vol. 30(5), pp. 961–81.
- Bell, D.E. (1985). 'Disappointment in decision making under uncertainty', *Operations Research*, vol. 33(1), pp. 1–27.
- Benjamin, D.J., Heffetz, O., Kimball, M.S. and Rees-Jones, A. (2014). 'Can marginal rates of substitution be inferred from happiness data? Evidence from residency choices', *American Economic Review*, (forthcoming).
- Bernoulli, D. (1738). 'Specimen theoriae novae de mensura sortis', *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, vol. 5, pp. 175–92. Translated into English by L. Sommer (1954) 'Exposition of a new theory on the measurement of risk', *Econometrica*, vol. 22(1), pp. 23–36. Reprinted in (A.N. Page 1968, ed.) *Utility Theory: Book of Readings*, ch. 11, New York: Wiley. Revised translation in (W.J. Baumol and S.M. Goldfeld 1968, eds.) *Precursors in Mathematical Economics: Anthology*, London: Clowes and Sons, Selection 2, pp. 15–26.
- Bikhchandani, S. and Segal, U. (2011). 'Transitive regret', *Theoretical Economics*, vol. 6(1), pp. 95–108.
- Birnbaum, M.H. (2008). 'New paradoxes of risky decision making', *Psychological Review*, vol. 115(2), pp. 463–501.
- Bleichrodt, H., Cillo, A. and Diecidue, E. (2010). 'A quantitative measurement of regret theory', *Management Science*, vol. 56(1), pp. 161–75.
- Bordalo, P., Gennaioli, N. and Shleifer, A. (2012). 'Salience theory of choice under risk', *Quarterly Journal of Economics*, vol. 127(4), pp. 1243–85.
- Bourgeois-Gironde, S. (2010). 'Regret and the rationality of choices', *Philosophical Transactions of the Royal Society B*, vol. 365(1), pp. 249–57.
- Bouyssou, D. and Pirlot, M. (2003). 'Nontransitive decomposable conjoint measurement', *Journal of Mathematical Psychology*, vol. 46(6), pp. 677–703.
- Braun, M. and Muermann, A. (2004). 'The impact of regret on the demand for insurance', *Journal of Risk and Insurance*, vol. 71(4), pp. 737–67.
- Budescu, D.V. and Weiss, W. (1987). 'Reflection of transitive and intransitive preferences: a test of prospect theory', *Organizational Behavior and Human Decision Processes*, vol. 39(2), pp. 184–202.
- Camerer, C.F. (1995). 'Individual decision making', in (J.H. Kagel and A.E. Roth eds.) *Handbook of Experimental Economics*, pp. 587–703, Princeton, NJ: Princeton University Press.

- Camille, N., Coricelli, G., Sallet, J., Pradat-Diehl, P., Duhamel, J.R. and Sirigu, A. (2004). 'The involvement of the orbitofrontal cortex in the experience of regret', *Science*, vol. 304(5647), pp. 1167–70.
- Chew, S.H. (1983). 'A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais paradox', *Econometrica*, vol. 51(4), pp. 1065–92.
- Chew, S.H. and MacCrimmon, K.R. (1979). 'Alpha-nu choice theory: an axiomatization of expected utility', Working Paper No. 669, Faculty of Commerce, University of British Columbia.
- Cubitt, R.P. and Sugden, R.F. (1998). 'The selection of preferences through imitation', *Review of Economic Studies*, vol. 65(4), pp. 761–71.
- Day, B. and Loomes, G. (2010). 'Conflicting violations of transitivity and where they may lead us', *Theory and Decision*, vol. 68(2), pp. 233–42.
- Delquié, P. and Cillo, A. (2006). 'Disappointment without prior expectation: a unifying perspective on decision under risk', *Journal of Risk and Uncertainty*, vol. 33(2), pp. 197–215.
- Diener, E. and Biswas-Diener, R. (2008). *Rethinking Happiness: The Science of Psychological Wealth*, Malden, MA: Blackwell Publishing.
- Engelbrecht-Wiggans, R. and Katok, E. (2008). 'Regret and feedback information in first-price sealed-bid auctions', *Management Science*, vol. 53(4), pp. 808–19.
- Filiz-Ozbay, E. and Ozbay, E.Y. (2007). 'Auctions with anticipated regret: theory and experiment', *American Economic Review*, vol. 97(4), pp. 1407–18.
- Fishburn, P.C. (1982). 'Nontransitive measurable utility', *Journal of Mathematical Psychology*, vol. 26(1), pp. 31–67.
- Fishburn, P.C. (1988). 'Expected utility: an anniversary and a new era', *Journal of Risk and Uncertainty*, vol. 1(3), pp. 267–83.
- Giorgetta, C., Grecucci, A., Bonini, N., Coricelli, G., Demarchi, G., Braun, C. and Sanfey, A.G. (2013). 'Waves of regret: a MEG study of emotion and decision-making', *Neuropsychologia*, vol. 51(1), pp. 38–51.
- Grether, D.M. and Plott, C.R. (1979). 'Economic theory of choice and the preference reversal phenomenon', *American Economic Review*, vol. 69(4), pp. 623–38.
- Harless, D.W. (1992). 'Actions versus prospects: the effect of problem representation on regret', *American Economic Review*, vol. 82(3), pp. 634–49.
- Hayashi, T. (2008). 'Regret aversion and opportunity dependence', *Journal of Economic Theory*, vol. 139(1), pp. 242–68.
- Howard, R.A., Matheson, J.E. and North, D.W. (1972). 'The decision to seed hurricanes', *Science*, vol. 176(June 16), pp. 1191–202.
- Humphrey, S.J. (1995). 'Regret aversion or event-splitting effects? More evidence under risk and uncertainty', *Journal of Risk and Uncertainty*, vol. 11(3), pp. 263–74.
- Humphrey, S.J. (2004). 'Feedback-conditional regret theory and testing regret aversion in risky choice', *Journal of Economic Psychology*, vol. 25(6), pp. 839–57.
- Iverson, G.I. and Falmagne, J.C. (1985). 'Statistical issues in measurement', *Mathematical Social Sciences*, vol. 10(2), pp. 131–53.
- Kahneman, D. (1994). 'New challenges to the rationality assumption', *Journal of Institutional and Theoretical Economics*, vol. 150(1), pp. 18–36.
- Kahneman, D. and Tversky, A. (1979). 'Prospect theory: an analysis of decision under risk', *Econometrica*, vol. 47(2), pp. 263–91.
- Kreweras, G. (1961). 'Sur une possibilité de rationaliser les intransitivités', *La Décision, Colloques Internationaux CNRS*, pp. 27–32.
- Laciana, C.E. and Weber, E.U. (2008). 'Correcting expected utility for comparisons between alternative outcomes: a unified parameterization of regret and disappointment', *Journal of Risk and Uncertainty*, vol. 36(1), pp. 1–17.
- Leland, J.W. (1994). 'Generalized similarity judgments: an alternative explanation for choice anomalies', *Journal of Risk and Uncertainty*, vol. 9(2), pp. 151–72.
- Leland, J.W. (1998). 'Similarity judgments in choice under uncertainty: a reinterpretation of the prediction of regret theory', *Management Science*, vol. 44(5), pp. 659–72.
- Lichtenstein, S. and Slovic, P. (1971). 'Reversals of preference between bids and choices in gambling decisions', *Journal of Experimental Psychology*, vol. 89(1), pp. 46–55.
- Lindman, H.R. (1971). 'Inconsistent preferences among gambles', *Journal of Experimental Psychology*, vol. 89(1), pp. 390–7.
- Lindman, H.R. and Lyons, J. (1978). 'Stimulus complexity and choice inconsistency among gambles', *Organizational Behavior and Human Decision Processes*, vol. 21(2), pp. 146–59.
- Loewenstein, G.F. and Ubel, P.A. (2008). 'Hedonic adaptation and the role of decision and experience utility in public policy', *Journal of Public Economics*, vol. 92(8–9), pp. 1795–810.
- Loomes, G. (1988a). 'When actions speak louder than prospects', *American Economic Review*, vol. 78(1), pp. 463–70.
- Loomes, G. (1988b). 'Further evidence of the impact of regret and disappointment in choice under uncertainty', *Economica*, vol. 55(1), pp. 47–62.

- Loomes, G. (1989). 'Predicted violations of the invariance principle in choice under uncertainty', *Annals of Operations Research*, vol. 19(1), pp. 103–13.
- Loomes, G. (2010). 'Modelling choice and valuation in decision experiments', *Psychological Review*, vol. 117(3), pp. 902–24.
- Loomes, G., Starmer, C. and Sugden, R.F. (1989). 'Preference reversal: information-processing effect of rational non-transitive choice?', *ECONOMIC JOURNAL*, vol. 99(Supplement), pp. 140–51.
- Loomes, G., Starmer, C. and Sugden, R.F. (1992). 'Are preferences monotonic: testing some implications of regret theory', *Economica*, vol. 59(233), pp. 17–33.
- Loomes, G. and Sugden, R.F. (1982). 'Regret theory: an alternative theory of rational choice under uncertainty', *ECONOMIC JOURNAL*, vol. 92(368), pp. 805–24.
- Loomes, G. and Sugden, R.F. (1987a). 'Testing for regret and disappointment in choice under uncertainty', *ECONOMIC JOURNAL*, vol. 97(Supplement), pp. 118–29.
- Loomes, G. and Sugden, R.F. (1987b). 'Some implications of a more general form of regret theory', *Journal of Economic Theory*, vol. 41, pp. 270–87.
- Loomes, G. and Sugden, R.F. (1998). 'Testing different stochastic specifications of risky choice', *Economica*, vol. 65, pp. 581–98.
- Loomes, G. and Taylor, C. (1992). 'Non-transitive preferences over gains and losses', *ECONOMIC JOURNAL*, vol. 102, pp. 357–65.
- Luce, R.D. and Raiffa, H. (1957). *Games and Decisions*, New York: Wiley.
- MacCrimmon, K.R. (1968). 'Descriptive and normative implications of the decision-theory postulates', in (K.H. Borch and J. Mossin, eds.) *Risk and Uncertainty*, pp. 3–23, New York: St. Martin's Press.
- Machina, M.J. (1982). 'Expected utility' analysis without the independence axiom', *Econometrica*, vol. 50(2), pp. 277–323.
- Mellers, B.A. and Biagini, K. (1994). 'Similarity and choice', *Psychological Review*, vol. 101(3), pp. 505–18.
- Michenaud, S. and Solnik, B. (2008). 'Applying regret theory to investment choices: currency hedging decisions', *Journal of International Money and Finance*, vol. 27(5), pp. 677–94.
- Moskowitz, H. (1974). 'Effects of problem representation and feedback on rational behavior in Allais and Morlat-type problems', *Decision Sciences*, vol. 5(2), pp. 225–42.
- Muermann, A., Mitchell, O.S. and Volkman, J.M. (2006). 'Regret, portfolio choice, and guarantees in defined contribution schemes', *Insurance: Mathematics and Economics*, vol. 39(2), pp. 219–29.
- Murray, R. and Beattie, J. (2001). 'Decisions about prenatal screening', in (E.U. Weber, J. Baron and G. Loomes, eds.) *Conflict and Tradeoffs in Decision Making*, pp. 156–74, Cambridge: Cambridge University Press.
- Perakis, G. and Roels, G. (2008). 'Regret in the newsvendor model with partial information', *Operations Research*, vol. 56(1), pp. 188–203.
- Quiggin, J. (1982). 'A theory of anticipated utility', *Journal of Economic Behaviour and Organization*, vol. 3(4), pp. 323–43.
- Regenwetter, M., Dana, J. and Davis-Stober, C.P. (2011). 'Transitivity of preferences', *Psychological Review*, vol. 118(1), pp. 42–56.
- Ritov, I. and Baron, J. (1990). 'Reluctance to vaccinate: omission bias and ambiguity', *Journal of Behavioral Decision Making*, vol. 3(4), pp. 263–77.
- Ritov, I. and Baron, J. (1995). 'Outcome knowledge, regret, and omission bias', *Organizational Behavior and Human Decision Processes*, vol. 64(2), pp. 119–27.
- Rubinstein, A. (1988). 'Similarity and decision-making under risk (is there a utility resolution to the Allais paradox?)', *Journal of Economic Theory*, vol. 46(1), pp. 145–53.
- Sarver, T. (2008). 'Anticipating regret: why fewer options may be better', *Econometrica*, vol. 76(2), pp. 263–305.
- Savage, L.J. (1951). 'The theory of statistical decision', *Journal of the American Statistical Association*, vol. 46(1), pp. 55–67.
- Savage, L.J. (1954). *The Foundations of Statistics*, New York: Wiley. (2nd ed. 1972, New York: Dover Publications.)
- Slovic, P. and Lichtenstein, S. (1983). 'Preference reversal: a broader perspective', *American Economic Review*, vol. 73(4), pp. 596–605.
- Smith, R.D. (1996). 'Is regret theory an alternative basis for estimating the value of health care interventions?', *Health Policy*, vol. 37(2), pp. 105–15.
- Starmer, C. (1992). 'Testing new theories of choice under uncertainty using the common consequence effect', *Review of Economic Studies*, vol. 59(4), pp. 813–30.
- Starmer, C. (1999). 'Cycling with rules of thumb: an experimental test for a new form of non-transitive behavior', *Theory and Decision*, vol. 46(2), pp. 141–58.
- Starmer, C. (2000). 'Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk', *Journal of Economic Literature*, vol. 38(2), pp. 332–82.
- Starmer, C. and Sugden, R.F. (1989). 'Probability and juxtaposition effects: an experimental investigation of the common ratio effect', *Journal of Risk and Uncertainty*, vol. 2(2), pp. 159–78.

- Starmer, C. and Sugden, R.F. (1993). 'Testing for juxtaposition and event-splitting effects', *Journal of Risk and Uncertainty*, vol. 6(1), pp. 235–54.
- Starmer, C. and Sugden, R.F. (1998). 'Testing alternative explanations of cyclical choices', *Economica*, vol. 65(259), pp. 347–61.
- Sugden, R.F. (1991). 'Rational choice: a survey of contributions from economics and philosophy', *ECONOMIC JOURNAL*, vol. 101(407), pp. 751–85.
- Sugden, R.F. (1993). 'An axiomatic foundation for regret theory', *Journal of Economic Theory*, vol. 60(1), pp. 159–80.
- Sugden, R.F. (2004). 'Alternatives to expected utility', in (S. Barberà, P.J. Hammond and C. Seidl, eds.), *Handbook of Utility Theory, vol. II*, pp. 685–755, Dordrecht: Kluwer Academic Publishers.
- Tversky, A. (1969). 'Intransitivity of preferences', *Psychological Review*, vol. 76(1), pp. 31–48.
- Tversky, A., Sattath, S. and Slovic, P. (1988). 'Contingent weighting in judgment and choice', *Psychological Review*, vol. 95(3), pp. 371–84.
- Tversky, A., Slovic, P. and Kahneman, D. (1990). 'The causes of preference reversal', *American Economic Review*, vol. 80(1), pp. 204–17.
- van Praag, B.M.S. and Ferrer-i-Carbonell, A. (2004). *Happiness Quantified*, Oxford: Oxford University Press.
- Vind, K. (2003). *Independence, Additivity, Uncertainty. With contributions by B. Grodal*, Berlin: Springer. (First version 1969).
- Wakker, P.P. (2010). *Prospect Theory: For Risk and Ambiguity*, Cambridge: Cambridge University Press.
- Wu, G. (1994). 'An empirical test of ordinal independence', *Journal of Risk and Uncertainty*, vol. 9(1), pp. 39–60.
- Zeelenberg, M. (1999). 'Anticipated regret, expected feedback and behavioral decision making', *Journal of Behavioral Decision Making*, vol. 12(1), pp. 93–106.
- Zeelenberg, M. and Pieters, R. (2007). 'A theory of regret regulation 1.0', *Journal of Consumer Psychology*, vol. 17(1), pp. 3–18.

Appendix B. Loomes, G. and Sugden, R. (1982). 'Regret theory: an alternative theory of rational choice under uncertainty', *ECONOMIC JOURNAL*, vol. 92(368), pp. 805–24.

REGRET THEORY: AN ALTERNATIVE THEORY OF RATIONAL CHOICE UNDER UNCERTAINTY*

Graham Loomes and Robert Sugden

The main body of current economic analysis of choice under uncertainty is built upon a small number of basic axioms, formulated in slightly different ways by von Neumann and Morgenstern (1947), Savage (1954) and others. These axioms are widely believed to represent the essence of rational behaviour under uncertainty. However, it is well known that many people behave in ways that systematically violate these axioms.¹

We shall initially focus upon a paper by Kahneman and Tversky (1979) which presents extensive evidence of such behaviour. Kahneman and Tversky offer a theory, which they call ‘prospect theory’, to explain their observations. We shall offer an alternative theory which is much simpler than prospect theory and which, we believe, has greater appeal to intuition.

The following notation will be used throughout. The *i*th *prospect* is written as X_i . If it offers increments or decrements of wealth x_1, \dots, x_n with probabilities p_1, \dots, p_n (where $p_1 + \dots + p_n = 1$) it may be denoted as $(x_1, p_1; \dots; x_n, p_n)$. Null consequences are omitted so that the prospect $(x, p; 0, 1 - p)$ is written simply as (x, p) . Complex prospects, i.e. those which offer other prospects as consequences, may be denoted as $(X_i, p_i; \dots; X_n, p_n)$. We shall use the conventional notation \succ, \succsim and \sim to represent the relations of strict preference, weak preference and indifference. We shall take it that for all prospects X_i and X_k , $X_i \succsim X_k$ or $X_i \preceq X_k$; but we shall not in general require that the relation \succsim is transitive.

I. KAHNEMAN AND TVERSKY'S EVIDENCE

Kahneman and Tversky's experiments offered hypothetical choices between pairs of prospects to groups of university faculty and students. Table 1 lists a selection of their results, which reveal three main types of violation of conventional expected utility theory:

- (a) The ‘certainty effect’ or ‘common ratio effect’, e.g. the conjunction of $X_5 \prec X_6$ and $X_9 \succ X_{10}$ and the conjunction $X_{13} \prec X_{14}$ and $X_{15} \succ X_{16}$. There is also a ‘reverse common ratio effect’, e.g. the conjunction of $X_7 \succ X_8$ and $X_{11} \prec X_{12}$.
- (b) The original ‘Allais Paradox’ or ‘common consequences effect’, e.g. the conjunction of $X_1 \prec X_2$ and $X_3 \succ X_4$.
- (c) The ‘isolation effect’ in two-stage gambles, e.g. the conjunction of $X_9 \succ X_{10}$ and $X_{17} \prec X_{18}$.

* We particularly wish to thank Michael Jones-Lee, Mark Machina and two anonymous referees for many helpful suggestions and criticisms.

¹ For a survey and discussion of much of the evidence, see Allais and Hagen (1979) and Schoemaker (1980, 1982).

Table 1 also reveals a 'reflection effect' where a change of sign on the consequences is associated with a reversal of the modal preference and the risk attitude that characterises it, e.g. $X_5 < X_6$ and $X_7 > X_8$. One instance of the reflection effect, revealed in Problems 14 and 14', may be interpreted as an

Table 1

Kahneman and Tversky problem no.	Prospects offered†	Modal preference	Percentage of subjects with modal preference	Characterisation of modal preference
1	$X_1 = (2,500, 0.33;$ $2,400, 0.66)$	$X_1 < X_2$	82*	Risk averse
2	$X_2 = (2,400, 1.00)$ $X_3 = (2,500, 0.33)$ $X_4 = (2,400, 0.34)$	$X_3 > X_4$	83*	Not clear
3	$X_5 = (4,000, 0.80)$ $X_6 = (3,000, 1.00)$	$X_5 < X_6$	80*	Risk averse
3'	$X_7 = (-4,000, 0.80)$ $X_8 = (-3,000, 1.00)$	$X_7 > X_8$	92*	Risk loving
4	$X_9 = (4,000, 0.20)$ $X_{10} = (3,000, 0.25)$	$X_9 > X_{10}$	65*	Not clear
4'	$X_{11} = (-4,000, 0.20)$ $X_{12} = (-3,000, 0.25)$	$X_{11} < X_{12}$	58	Not clear
7	$X_{13} = (6,000, 0.45)$ $X_{14} = (3,000, 0.90)$	$X_{13} < X_{14}$	86*	Risk averse
8	$X_{15} = (6,000, 0.001)$ $X_{16} = (3,000, 0.002)$	$X_{15} > X_{16}$	73*	Risk loving
10	$X_{17} = (X_5, 0.25)$ $X_{18} = (X_6, 0.25)$	$X_{17} < X_{18}$	78*	Risk averse
14	$X_{19} = (5,000, 0.001)$ $X_{20} = (5, 1.000)$	$X_{19} > X_{20}$	72*	Risk loving
14'	$X_{21} = (-5,000, 0.001)$ $X_{22} = (-5, 1.000)$	$X_{21} < X_{22}$	83*	Risk averse

* Statistically significant at the 0.01 level.

† Consequences are increments or decrements of wealth, measured in Israeli pounds.

example of simultaneous gambling and insurance, since $X_{19} > X_{20}$ indicates a willingness to enter an actuarially fair lottery offering a small probability of a large prize, while $X_{21} < X_{22}$ signifies a willingness to take out actuarially fair insurance against a small probability of a large loss. We also note an interesting mixture of risk attitudes. Sometimes risk aversion is associated with problems involving increments of wealth, e.g. $X_{13} < X_{14}$, and sometimes with problems involving decrements, e.g. $X_{21} < X_{22}$. Likewise, risk loving is sometimes associated with problems involving increments, e.g. $X_{15} > X_{16}$, and sometimes with problems involving decrements, e.g. $X_7 > X_8$.

Simultaneous gambling and insurance, the reflection effect, and the mixture of risk attitudes may all be accommodated by conventional expected utility theory, though only at the cost of certain fairly arbitrary assumptions and some rather unsatisfactory implications.¹ But no accommodation is possible for the effects listed in (a), (b) and (c) above – the observations here simply violate one or more of the conventional¹ axioms.

¹ See Friedman and Savage (1948), Markowitz (1952) and Hirschleifer (1966).

However, in the next section we shall outline the framework of an alternative theory which not only explains the reflection effect and simultaneous gambling and insurance, but also predicts the behaviour described in (a), (b) and (c). We shall then argue that, besides being predictable, such behaviour can be defended as rational, and that our model therefore provides the basis for an alternative theory of rational choice under uncertainty.

II. THE FRAMEWORK OF AN ALTERNATIVE THEORY

We consider an individual in a situation where there is a finite number, n , of alternative *states of the world*, any one of which might occur. Each state j has a probability p_j where $0 < p_j \leq 1$ and $p_1 + \dots + p_n = 1$. These probabilities may be interpreted either as objective probabilities known to the individual or, in the absence of firm knowledge of this kind, as subjective probabilities which represent the individual's degree of belief or confidence in the occurrence of the corresponding states. The individual's problem is to choose between *actions*. Each action is an n -tuple of *consequences*, one consequence for each state of the world. We shall write the consequence of the i th action in the event that the j th state occurs as x_{ij} . Consequences need not take the form of changes in wealth, although in our applications of our theory, we shall interpret x_{ij} as an increment or decrement of wealth, measured relative to some arbitrary level (which need not be the individual's current wealth). Notice that actions, unlike prospects, associate consequences with particular states of the world. Thus a number of different actions might correspond with the same prospect. We shall recognise this difference by using the symbol A for actions, reserving X for prospects. Thus far, our theory has a close resemblance to Savage's, except in that we take probabilities as given, just as von Neumann and Morgenstern do.

A choice problem may involve any number of available actions, but we shall begin by analysing problems where there is only a pair of actions to choose between. All of Kahneman and Tversky's evidence concerns the behaviour of people choosing between pairs of prospects. Choices between three or more actions raise some additional issues, which we shall discuss in Section IV.

Our first assumption is that for any given individual there is a *choiceless utility function* $C(\cdot)$, unique up to an increasing linear transformation, which assigns a real-valued utility index to every conceivable consequence. The significance of the word 'choiceless' is that $C(x)$ is the utility that the individual would derive from the consequence x if he experienced it *without having chosen it*. For example, he might have been compelled to have x by natural forces, or x might have been imposed on him by a dictatorial government. Thus – in contrast to the von Neumann–Morgenstern concept of utility – our concept of choiceless utility is defined independently of choice. Our approach is utilitarian in the classical sense. What we understand by 'choiceless utility' is essentially what Bernoulli and Marshall understood by 'utility' – the psychological experience of pleasure that is associated with the satisfaction of desire. We believe that it is possible to introspect about utility, so defined, and that it is therefore meaningful to talk about utility being experienced in choiceless situations.

Now suppose that an individual experiences a particular consequence as the result of an act of choice. Suppose that he has to choose between actions A_1 and A_2 in a situation of uncertainty. He chooses A_1 and then the j th state of the world occurs. He therefore experiences the consequence x_{1j} . He now knows that, had he chosen A_2 instead, he would be experiencing x_{2j} . Our introspection suggests to us that the psychological experience of pleasure associated with having the consequence x_{1j} in these circumstances will depend not only on the nature of x_{1j} but also on the nature of x_{2j} . If x_{2j} is a more desirable consequence than x_{1j} , the individual may experience *regret*: he may reflect on how much better his position would have been, had he chosen differently, and this reflection may reduce the pleasure that he derives from x_{1j} . Conversely, if x_{1j} is the more desirable consequence, he may experience what we shall call *rejoicing*, the extra pleasure associated with knowing that, as matters have turned out, he has taken the best decision.

We guess that many readers will recognise these experiences. For example, compare the sensation of losing £100 as the result of an increase in income tax rates, which you could have done nothing to prevent, with the sensation of losing £100 on a bet on a horse race. Our guess is that most people would find the latter experience more painful, because it would inspire regret. Conversely, compare the experience of gaining £100 from an income tax reduction with that of winning £100 on a bet. Now we should guess that most people would find the latter experience more pleasurable. This concept of regret resembles Savage's (1951) notion in some ways, but it will emerge that our theory is very different from his minimax regret criterion.

We shall incorporate the concepts of regret and rejoicing into our theory by means of a *modified utility function*. Suppose that an individual chooses action A_i in preference to action A_k , and that the j th state of the world occurs. The actual consequence is x_{ij} while, had he chosen differently, x_{kj} would have occurred. We shall write $C(x_{ij})$ as c_{ij} and we shall then say that the individual experiences the *modified utility* m_{ij}^k where:

$$m_{ij}^k = M(c_{ij}, c_{kj}). \quad (1)$$

The function $M(\cdot)$ assigns a real-valued index to every ordered pair of choiceless utility indices. The difference between m_{ij}^k and c_{ij} may be interpreted as an increment or decrement of utility corresponding with the sensations of rejoicing or regret. To formulate regret and rejoicing in this way is to assume that the degree to which a person experiences these sensations depends only on the choiceless utility associated with the two consequences in question – ‘what is’ and ‘what might have been’ – and is independent of any other characteristics of these consequences. Given this assumption, it is natural to assume in addition that if $c_{ij} = c_{kj}$ then $m_{ij}^k = c_{ij}$: if what occurs is exactly as pleasurable as what might have occurred, there is neither regret nor rejoicing. It is equally natural to assume that $\partial m_{ij}^k / \partial c_{kj} \leq 0$: the more pleasurable the consequence that might have been, the more regret – or less rejoicing – is experienced. (We include as a limiting case the possibility that a person might not experience regret or rejoicing at all.) We also make the uncontroversial assumption that $\partial m_{ij}^k / \partial c_{ij} > 0$: that, other things being equal, modified utility increases with choiceless utility.

Our theory is that the individual chooses between actions so as to maximise the mathematical expectation of modified utility. We may define the *expected modified utility* E_i^k of action A_i , evaluated with respect to action A_k , by:

$$E_i^k = \sum_{j=1}^n p_j m_{ij}^k. \tag{2}$$

Faced with a choice between A_i and A_k , the individual will prefer A_i , prefer A_k or be indifferent between them according to whether E_i^k is greater than, less than or equal to E_k^i .

Why, it may be asked, do we assume that people maximise the mathematical expectation of modified utility? Principally because this is a simple assumption which yields implications consistent with empirical evidence. We do not claim that maximising expected modified utility is the only objective that is consistent with a person being rational. However – and we shall say more about this in Section V – we believe that this is not *irrational*, and that, given the utilitarian premises of our approach, there is at least a presumption that people who experience regret and rejoicing will seek to maximise expected modified utility. Notice that, in our theory, someone who does not feel regret or rejoicing at all will simply maximise expected choiceless utility. This special case of our theory corresponds with expected utility theory in its traditional or Bernoullian form, in which utility is interpreted as a psychological experience. To assume that people maximise expected modified utility is to generalise Bernoulli’s theory in a very natural way, since the individual who *does* experience rejoicing and regret can be expected to try to anticipate those feelings and take them into account when making a decision under uncertainty.

We shall now show that all of the experimental evidence described in Section I is consistent with regret theory. We shall do this by taking a restricted form of our general theory and by showing that the experimental evidence is consistent with this restricted form.

The particular restriction involves a simplifying assumption about the function $M(\cdot)$. We shall assume that the degree of regret or rejoicing that a person experiences depends only on the difference between the choiceless utility of ‘what is’ and the choiceless utility of ‘what might have been’. This allows us to define a *regret-rejoice function* $R(\cdot)$ which assigns a real-valued index to every possible increment or decrement of choiceless utility, and then to write:

$$m_{ij}^k = c_{ij} + R(c_{ij} - c_{kj}). \tag{3}$$

It follows from the assumptions we have made about $M(\cdot)$ that $R(0) = 0$ and that $R(\cdot)$ is non-decreasing. In the limiting case in which $R(\xi) = 0$ for all ξ , regret theory would yield exactly the same predictions as expected utility theory. Since we wish to emphasise the differences between the two theories we shall assume that $R(\cdot)$ is strictly increasing and three times differentiable.

Now suppose, as before, that an individual has to choose between the actions A_i and A_k . The individual will have the weak preference $A_i \succcurlyeq A_k$ if and only if:

$$\sum_{j=1}^n p_j [c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geq 0. \tag{4}$$

It is convenient to define a function $Q(\cdot)$ such that for all ξ ,

$$Q(\xi) = \xi + R(\xi) - R(-\xi). \quad (5)$$

Thus $A_i \geq A_k$ if and only if:

$$\sum_{j=1}^n p_j [Q(c_{ij} - c_{kj})] \geq 0. \quad (6)$$

$Q(\cdot)$ is an increasing function which has the following property of symmetry: for all ξ , $Q(\xi) = -Q(-\xi)$. Thus to know the value of $Q(\xi)$ for all $\xi \geq 0$ is to know the value of $Q(\xi)$ for all ξ .

Three alternative simplifying assumptions about $Q(\cdot)$ can be distinguished:

Assumption 1. $Q(\cdot)$ is linear or equivalently, for all ξ , $R''(\xi) = R''(-\xi)$. It follows immediately from (6) that in this case the individual will behave exactly as if he were maximising expected choiceless utility. Thus regret theory would yield the same predictions as expected utility theory and choiceless utility indices would be operationally indistinguishable from von Neumann–Morgenstern utility indices.

Assumption 2. $Q(\cdot)$ is concave for all positive values of ξ or equivalently, for all $\xi > 0$, $R''(\xi) < R''(-\xi)$.

Assumption 3. $Q(\cdot)$ is convex for all positive values of ξ or equivalently, for all $\xi > 0$, $R''(\xi) > R''(-\xi)$.

On the face of it, there seems to be no *a priori* reason for preferring any one of these assumptions to the others. They are simply alternative assumptions about human psychology and a choice between them should be made mainly on the basis of empirical evidence.¹ We shall therefore show that all the evidence listed in Table 1 is consistent with the restricted form of our theory under Assumption 3. In contrast, Assumption 1 would predict no violations of expected utility theory, while Assumption 2 would predict violations, but in the opposite direction to those generally observed.

III. SOME IMPLICATIONS OF REGRET THEORY

We shall now derive some implications of our theory concerning choices between pairs of *statistically independent* prospects. In our theory, a choice problem cannot be analysed unless a matrix of state-contingent consequences can be specified, and a given pair of prospects (i.e. probability distributions of consequences) may

¹ We say 'mainly' because there may be some theoretical reasons for expecting Assumption 3 to be true more often than either of the other two assumptions. Notice that it is sufficient (but not a necessary) condition for Assumption 1 to hold that, for all ξ , $R'''(\xi) = 0$. Similarly it is sufficient for Assumption 2 to hold that, for all ξ , $R'''(\xi) < 0$; and it is sufficient for Assumption 3 to hold that, for all ξ , $R'''(\xi) > 0$. Consider the following three alternative cases: that $R(\cdot)$ is linear, that it is everywhere convex, and that it is everywhere concave. Linearity entails that for all ξ , $R'''(\xi) = 0$ and so entails Assumption 1. Convexity entails that for all ξ , $R'(\xi) > 0$ and $R''(\xi) > 0$. Since, given these two conditions, $R'''(\xi) \leq 0$ cannot hold for all ξ , the simplest assumption to make about $R'''(\cdot)$ is that for all ξ , $R'''(\xi) > 0$. This in turn entails Assumption 3. Concavity entails that for all ξ , $R'(\xi) > 0$ and $R''(\xi) < 0$. Since $R'''(\xi) \leq 0$ cannot hold for all ξ , the simplest assumption to make is again that for all ξ , $R'''(\xi) > 0$. So Assumption 3 fits with both convexity and concavity, while Assumption 1 is appropriate only for linearity – which is only one point on a continuous spectrum which ranges from extreme convexity to extreme concavity.

be capable of being represented by many different matrices. However, the assumption of statistical independence ensures that there is a unique matrix for each pair of prospects. In most of Kahneman and Tversky's experiments, subjects were simply asked to choose between pairs of prospects. In such cases, we suggest, the most natural assumption for subjects to make is that the prospects are independent. Given this assumption, we can show that the evidence of Table 1 is entirely consistent with regret theory. As before, we shall use x_1 and x_2 to represent consequences. We shall use c_1 and c_2 to represent the choiceless utility indices $C(x_1)$ and $C(x_2)$. For simplicity, we choose a transformation of $C(\cdot)$ such that $C(0) = 0$; and we assume that $C(\cdot)$ is an increasing function.

(a) *The 'common ratio effect', and its reverse*

Our theory yields the following prediction, which violates expected utility theory:

Let $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$ be independent prospects, where $1 \geq p > 0$ and $1 > \lambda > 0$. If there exists some probability \bar{p} such that $X_i \sim X_k$ when $p = \bar{p}$, then (i) *(the common ratio effect)* if $x_1 > x_2 > 0$, then $p < \bar{p} \Rightarrow X_i \succ X_k$ and $p > \bar{p} \Rightarrow X_i \prec X_k$ and (ii) *(the reverse common ratio effect)* if $0 > x_2 > x_1$, then $p < \bar{p} \Rightarrow X_i \prec X_k$ and $p > \bar{p} \Rightarrow X_i \succ X_k$.

In proving this result, it is convenient to begin by stating a general property of our theory. Let $X' = (x_1, p_1)$ and $X'' = (x_2, p_2)$ be any two independent prospects. The choice between these prospects may be represented by the matrix given in

Table 2

Action corresponding with prospect	$p_1 p_2$	$p_1(1-p_2)$	$(1-p_2)p_2$	$(1-p_1)(1-p_2)$
X'	x_1	x_1	0	0
X''	x_2	0	x_2	0

Table 2, where each column represents a different state of the world, and the probability that each state will occur is given at the top of its column. Applying Expression (6) to Table 2, we find: that

$$X' \succsim X'' \text{ iff } p_1 Q(c_1) - p_2 Q(c_2) - p_1 p_2 [Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \geq 0. \quad (7)$$

Thus in the case where $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$,

$$X_i \succsim X_k \text{ iff } p\{\lambda Q(c_1) - Q(c_2) - \lambda p[Q(c_1) - Q(c_1 - c_2) - Q(c_2)]\} \geq 0. \quad (8)$$

By assumption, $Q(c)$ is convex for all $c > 0$ so that when $c_1 > c_2 > 0$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] > 0$. Given this inequality, the common ratio effect follows straightforwardly from Expression (8). Conversely, when $0 > c_2 > c_1$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] < 0$; and this implies the reverse common ratio effect.

The evidence of Problems 3 and 4 is consistent with the existence of the common ratio effect. Let $x_1 = 4,000$, $x_2 = 3,000$ and $\lambda = 0.8$. Then if $p = 1.0$, X_5

= $(x_1, \lambda p)$ and $X_6 = (x_2, p)$. If $p = 0.25$, $X_9 = (x_1, \lambda p)$ and $X_{10} = (x_2, p)$. The conjunction of preferences $X_5 < X_6$ and $X_9 > X_{10}$ violates expected utility theory but is consistent with regret theory (corresponding with the case $1.0 > \bar{p} > 0.25$). Over half of Kahneman and Tversky's subjects had this conjunction of preferences. Further evidence of the common ratio effect is provided by Problems 7 and 8, while Problems 3' and 4' reveal the reverse common ratio effect.

(b) *The 'common consequences effect' or Allais paradox*

Our theory yields a further prediction, which also violates expected utility theory:

Let $X_i = (x_1, p_1; x_2, \alpha)$ and $X_k = (x_2, p_2 + \alpha)$ be independent prospects where $1 \geq p_2 > p_1 > 0$ and $(1 - p_2) \geq \alpha \geq 0$. If there exists some probability $\bar{\alpha}$ such that $X_i \sim X_k$ when $\alpha = \bar{\alpha}$, then (i) (*the common consequences effect*) if $x_1 > x_2 > 0$, then $\alpha < \bar{\alpha} \Rightarrow X_i > X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i < X_k$ and (ii) (*the reverse common consequences effect*) if $0 > x_2 > x_1$, then $\alpha < \bar{\alpha} \Rightarrow X_i < X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i > X_k$.

According to regret theory,

$$X_i \begin{matrix} \succ \\ \simeq \\ \prec \end{matrix} X_k \text{ iff } p_1 Q(c_1) - p_2 Q(c_2) - p_1(p_2 + \alpha) [Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \begin{matrix} \geq \\ = \\ \leq \end{matrix} 0. \quad (9)$$

Because $Q(c)$ is assumed to be convex for all $c > 0$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)]$ is positive if $x_1 > x_2 > 0$ and negative if $0 > x_2 > x_1$. Given these two propositions, Expression (9) entails both the common consequences effect and the reverse common consequences effect.

The evidence of Problems 1 and 2 is consistent with the existence of the common consequences effect. Let $x_1 = 2,500$, $x_2 = 2,400$, $p_1 = 0.33$ and $p_2 = 0.34$. Then if $\alpha = (1 - p_2)$, $X_1 = (x_1, p_1; x_2, \alpha)$ and $X_2 = (x_2, p_2 + \alpha)$. If $\alpha = 0$, $X_3 = (x_1, p_1; x_2, \alpha)$ and $X_4 = (x_2, p_2 + \alpha)$. The conjunction of preferences $X_1 < X_2$ and $X_3 > X_4$ violates expected utility theory but is consistent with regret theory (corresponding with the case $0.66 > \bar{\alpha} > 0$). At least 65% of Kahneman and Tversky's subjects had this conjunction of preferences. Kahneman and Tversky did not publish any results relevant to our prediction of a reverse common consequences effect.

(c) *The 'isolation effect' in the two-stage gambles*

In Kahneman and Tversky's Problem 10, their respondents were offered a two-stage gamble. In the first stage there was a 0.75 probability of the gamble ending with a null consequence and a 0.25 probability of going through to the second stage. Before embarking on the first stage, respondents were asked to choose which of X_5 or X_6 they would prefer if they got through to the second stage.

According to the compound probability axiom of expected utility theory, $X_{17} = (X_5, 0.25)$ is equivalent to $(4,000, 0.20)$ which is simply prospect X_9 ; and $X_{18} = (X_6, 0.25)$ is equivalent to $(3,000, 0.25)$ which is prospect X_{10} . Thus expected utility theory makes no distinction between Problem 10 and Problem 4.

However, regret theory does make a distinction. The simple prospects X_9 and X_{10} are regarded as statistically independent, and Problem 4 is therefore represented by the matrix of state-contingent consequences shown in Table 3a. By

contrast, prospects X_{17} and X_{18} are not statistically independent: the first stage of the gamble is common to both, and if the state occurs under which the gamble comes to an end, the individual receives the same null consequence whichever prospect was chosen. Hence Problem 10 is represented by the matrix of state-contingent consequences shown in Table 3*b*. Since Tables 3*a* and 3*b* are

Table 3*a*

Action corresponding with prospect	0.60	0.20	0.15	0.05
X_9	0	0	4,000	4,000
X_{10}	0	3,000	0	3,000

Table 3*b*

Action corresponding with prospect	0.75	0.20	0.05
X_{17}	0	4,000	0
X_{18}	0	3,000	3,000

different, our theory provides no reason to suppose that an individual will have the same preferences between X_{17} and X_{18} as between X_9 and X_{10} .

Before analysing this example further, we present a result which holds for regret theory in its most general form, and which we shall call the *separability principle*.

Let S_1, \dots, S_n be mutually exclusive events (i.e. non-intersecting sets of states of the world) with the non-zero probabilities p_1, \dots, p_n where $p_1 + \dots + p_n = 1$. Let S'_1, \dots, S'_{n+1} be mutually exclusive events with the probabilities $\mu p_1, \dots, \mu p_n, 1 - \mu$, where $0 < \mu < 1$. Let $A_i = (x_{11}, \dots, x_{1n})$ and $A_k = (x_{21}, \dots, x_{2n})$ be any two actions defined in relation to the events S_1, \dots, S_n . Let A_a and A_b be actions defined in relation to the events S'_1, \dots, S'_{n+1} , such that $A_a = (x_{11}, \dots, x_{1n}, y)$ and $A_b = (x_{21}, \dots, x_{2n}, y)$, y being any consequence common to both actions. Then $A_a \succcurlyeq A_b$ if and only if $A_i \succcurlyeq A_k$.

The proof is straightforward. If E_i^k and E_k^i are the expected modified utilities of A_i and A_k , evaluated in relation to one another, then $E_a^b = \mu E_i^k + (1 - \mu) C(y)$ and $E_b^a = \mu E_k^i + (1 - \mu) C(y)$. Hence $E_i^k \geq E_k^i \Leftrightarrow E_a^b \geq E_b^a$, which entails $A_i \succcurlyeq A_k \Leftrightarrow A_a \succcurlyeq A_b$. The separability principle entails Savage's sure-thing principle as a special case. Let μ remain constant, and let us construct two new actions, A_c and A_d , which are the same as A_a and A_b except that the common consequence y is replaced by the common consequence z . It is clear that $A_i \succcurlyeq A_k \Leftrightarrow A_c \succcurlyeq A_d$, and hence it follows that $A_a \succcurlyeq A_b \Leftrightarrow A_c \succcurlyeq A_d$, which is Savage's sure-thing principle.

Returning to Kahneman and Tversky's evidence, let A_5 and A_6 be the actions corresponding to the independent prospects X_5 and X_6 , and let A_{17} and A_{18} be

the actions corresponding to X_{17} and X_{18} in Table 3 *b*. Since $E_{17}^{18} = \mu E_5^6 + (1 - \mu) C(0)$ and $E_{18}^{17} = \mu E_6^5 + (1 - \mu) C(0)$, it follows that $X_5 < X_6 \Leftrightarrow X_{17} < X_{18}$. We have already seen in (a) above that the conjunction $X_5 < X_6$ and $X_9 > X_{10}$ is consistent with our theory. Thus it follows that the conjunction $X_9 > X_{10}$ and $X_{17} < X_{18}$, which violates conventional expected utility, is also consistent with regret theory.

(d) *The 'reflection effect'*

The results in (a), (b) and (c) above were derived without making any assumption about $C(\cdot)$ other than that it is monotonically increasing. We shall derive our results in (d) and (e) by making the additional assumption that $C(\cdot)$ is linear; and, for convenience, we shall choose a transformation of that linear function such that for all x , $C(x) = x$.

Consider two independent prospects, $X_i = (x_1, p_1)$ and $X_k = (x_2, p_2)$. Their 'reflections' are denoted $X'_i = (-x_1, p_1)$ and $X'_k = (-x_2, p_2)$. From Expression (7) we know that $X_i \succcurlyeq X_k$ if and only if:

$$p_1 Q(x_1) - p_2 Q(x_2) - p_1 p_2 [Q(x_1) - Q(x_1 - x_2) - Q(x_2)] \geq 0. \quad (10)$$

Now exactly the same inequality is necessary and sufficient for $X'_i \preccurlyeq X'_k$. Hence $X_i \succcurlyeq X_k \Leftrightarrow X'_i \preccurlyeq X'_k$. Thus if $C(\cdot)$ is linear, the reflection effect is always observed.

Our intuition is that $C(\cdot)$ is not linear but concave. If this is correct, the reflection effect will not always be observed, and in particular, individuals will reject actuarially fair 50-50 gambles, rather than being indifferent towards them. This point is discussed further in Section V.

(e) *Mixed risk attitudes; simultaneous gambling and insurance*

Consider two independent prospects which offer an actuarially fair gamble: $X_i = (0, 1)$ and $X_k = (x, p; -px/(1-p), 1-p)$, where $0 < p < 1$ and $x > 0$. Maintaining our previous assumption about $C(\cdot)$ we can apply Expression (7) and rearrange to give:

$$X_i \succcurlyeq X_k \text{ iff } Q\left(\frac{px}{1-p}\right) - \frac{p}{1-p} Q(x) \equiv 0. \quad (11)$$

From the assumption that $Q(x)$ is convex for all $x > 0$, it follows that $X_i \succcurlyeq X_k$ as $p \equiv 0.5$. So the individual will accept small-stake large-prize fair gambles ($p < 0.5$) but reject large-stake small-prize fair gambles ($p > 0.5$). Insurance typically involves paying a small premium to avoid a small probability of a large loss; thus in terms of our theory – which does not use the concept of a 'reference point' – to buy actuarially fair insurance is to reject a large-stake small-prize fair gamble, and thus it is consistent with our theory that an individual may simultaneously insure and accept small-stake large-prize gambles. Moreover, we can construct both small-stake large-prize fair gambles, and large-stake small-prize fair gambles either with all consequences positive or with all consequences negative. Thus a mixture of risk attitudes in both the positive and the negative domain is also consistent with our theory.

These conclusions would require some modification if $C(\cdot)$ were assumed to be concave rather than linear. In this case it can be shown that $X_i \succ X_k$ if $p \geq 0.5$,

but it is no longer possible to make a firm prediction when $p < 0.5$. However, if an individual is more strongly influenced by the shape of $Q(\cdot)$ than by the non-linearity of $C(\cdot)$, simultaneous gambling and insurance is still consistent with our theory.

IV. TRANSITIVITY OF PREFERENCES AND MULTI-ACTION PROBLEMS

One controversial property of our theory is that \succcurlyeq , the relation of weak preference, is not necessarily transitive. Consider the three actions shown in Table 4 in relation to an individual for whom $C(\cdot)$ is linear. Relative to A_1 , A_2 is a large-stake small-prize fair gamble, so that the individual would have the preference $A_1 \succ A_2$ if he had to choose between these two actions. If, as our theory entails, the individual acts according to the separability principle outlined in Section III (c), state S_1 can be ignored in a comparison between A_2 and A_3 . Thus, relative to A_2 , A_3 is also a large-stake small-prize fair gamble, and so $A_2 \succ A_3$. However, relative to A_1 , A_3 is a small-stake large-prize fair gamble, so that $A_3 \succ A_1$. This is not to say that our theory specifically predicts non-transitive pairwise choices (since the $C(\cdot)$ function need not be linear); but such choices can be consistent with the theory.

Table 4

Action	S_1 0.4	S_2 0.2	S_3 0.4
A_1	6	6	6
A_2	0	10	10
A_3	0	0	15

The example shows that an individual will necessarily make non-transitive choices if (i) he acts according to the separability principle (or according to the sure-thing principle), (ii) he always accepts small-stake large-prize fair gambles and (iii) he always rejects large-stake small-prize fair gambles. In the light of the evidence that many people simultaneously gamble and insure one might well argue that a satisfactory theory of choice under uncertainty should encompass the case of the individual who acts according to (ii) and (iii). To say this is to say that either the sure-thing principle or the axiom of transitivity must be dropped. Our theory differs from many of its rivals by dropping transitivity rather than the sure-thing principle.

This raises two questions. One is whether a theory that allows non-transitive pairwise choices can be regarded as a theory of rational behaviour; this issue is discussed in Section V. The other question is how to extend our theory to deal with multi-action choice problems: since in our theory the relation \succcurlyeq is not necessarily transitive, we cannot deal with choices from sets of three or more actions simply by invoking the idea of a preference ordering. We shall argue that the logic of regret and rejoicing points towards a different way of generalising a theory of pairwise choice.

Consider the problem of choosing one action from a set S . The logic of our approach requires that the individual should evaluate each action in turn by asking himself what sensations of regret or rejoicing he would experience in each state of the world, were he to choose that action. Since to choose one action is to reject all of the others, the individual could experience regret or rejoicing in contemplating any of the rejected actions. This idea might be formulated in the following way. As before, we use E_i^k to represent the expected modified utility of choosing action A_i in a situation where the only alternative is action A_k . Now let E_i^S represent the expected modified utility of choosing A_i from the set of actions S . It seems natural to make E_i^S a weighted average of the values of E_i^k for each of the actions A_k in S (other than A_i itself). One way of building this idea into our theory would be to assign *action weights* a_k^S to each action A_k in S , normalised so that these weights sum to unity. Then E_i^S could be defined as:

$$E_i^S = \sum_{k \in S} \frac{a_k^S}{1 - a_i^S} E_i^k \quad (k \neq i). \tag{12}$$

The individual’s decision rule, as in the case of pairwise choice, would be to maximise expected modified utility. We hope in the future to formulate a theory of action weights, but in the example which follows we shall just make the simplest assumption – that each action has the same weight.

Table 5

Action	1/3	1/3	1/3
A_1	1	1	1
A_2	0	0	3
A_3	0	3	0

This illustrative example refers to the choice problem shown in Table 5. As before, we shall assume that $C(x) = x$, and we shall make a particular assumption about the regret–rejoice function, that over the relevant range, $R(\xi) = 1 - 0.8\xi^5$. In this case, and for these three actions, the relation \succcurlyeq happens to be transitive; $A_2 \succ A_1$, $A_3 \succ A_1$, $A_2 \sim A_3$. It is tempting (but, we suggest, wrong) to conclude from this that A_1 will not be chosen from the set $\{A_1, A_2, A_3\}$. If the action weights are equal to one another then $E_1^S = 0.946$, $E_2^S = 0.899$ and $E_3^S = 0.899$, so that, according to the decision rule, A_1 will be chosen. Whether or not such behaviour can be defended as rational will be discussed in Section V.

V. THE POSITIVE AND NORMATIVE STATUS OF REGRET THEORY

The experimental results published by Kahneman and Tversky, wide-ranging though they are, form only a small fraction of the evidence accumulated in the past 30 years to show consistent and repeated violations of certain axioms of expected utility theory. Regret theory is one of a number of alternative theories that have been proposed in the light of this evidence; other theories have been

presented by, for example, Allais (1953), Kahneman and Tversky (1979), Fishburn (1981) and Machina (1982). We shall shortly compare our theory with these others, but first let us discuss a possible argument against regret theory.

It might be objected that regret theory is limited to cases where probabilities are known, and that it rests on assumptions about non-observable functions, whereas expected utility theory is built on clear behavioural axioms which make it possible, in principle, to construct a series of choice problems which will reveal the individual's von Neumann–Morgenstern utility function.

While we do not share the methodological position that the only satisfactory theories are those formulated entirely in terms of empirical propositions, we would point out that if an individual behaves according to our model, it is possible in principle to infer from observations of his choices: his subjective probabilities; his $C(\cdot)$ function (unique up to a positive linear transformation); and his $Q(\cdot)$ function (which, for any given transformation of $C(\cdot)$, will be unique up to a positive linear transformation with a fixed point at the origin). Thus each of the assumptions about $C(\cdot)$ and $Q(\cdot)$ required to generate our predictions is in principle capable of empirical refutation. (For an outline of the procedures involved, see the Appendix.)

The other criteria that are commonly used to evaluate positive theories are predictive power, simplicity and generality. Regret theory yields a wide range of firm predictions that are supported by experimental evidence, and it does so on the basis of a remarkably simple structure. Only the two functions $C(\cdot)$ and $Q(\cdot)$ are required. As far as $C(\cdot)$ is concerned, some of the most important predictions of our model – the common ratio effect, the common consequences effect, their reverses, and the isolation effect – require only that this function is monotonically increasing; the additional assumption of linearity yields clear predictions concerning the reflection effect and simultaneous gambling and insurance. In generating all these predictions, the other crucial assumption is simply that $Q(\xi)$ is convex for all $\xi > 0$.

Thus in comparison with Kahneman and Tversky's 'prospect theory' – which is also consistent with all the evidence in Table 1 – regret theory is very simple indeed. Kahneman and Tversky's theory superimposes on expected utility theory a theory of systematic violations. Among their many assumptions are: (i) the rounding of probabilities up or down, and the complete editing out of 'small' probabilities; (ii) a 'decision weight function' which overweights small probabilities, underweights large probabilities, involves 'subcertainty', 'sub-proportionality' and 'subadditivity', and which is discontinuous at both ends, thus implying certain 'quantal effects'; and (iii) a 'value function' (essentially a utility function) which *must* have at least one point of inflection (at the individual's 'reference point' – which may or may not move around) but which can, if required, have no less than five points of inflection. We believe that against the complex and somewhat ad hoc array of assumptions required by prospect theory the principle of Occam's Razor strongly favours the straightforwardness of regret theory.

Allais's and Machina's theories are considerably simpler than prospect theory, but they cannot explain all of the evidence in Table 1. Both of these theories

assume that the individual has a preference ordering over prospects. Thus two of the fundamental principles of expected utility theory are retained: that pairwise choices are transitive and that courses of action associated with identical probability distributions of consequences are equivalent to one another. (We shall call this latter principle the *equivalence axiom*.) Allais and Machina break away from expected utility theory by dropping the independence axiom; given that the equivalence axiom is retained, this amounts to abandoning the sure-thing principle. Our strategy is radically different: we retain the sure-thing principle while jettisoning both the equivalence axiom and the transitivity axiom. As a result we are able to predict the isolation effect in two-stage gambles, a form of observed behaviour that contravenes the equivalence axiom and therefore cannot be explained by either Allais or Machina. We are also able to predict the systematic occurrence of the reflection effect. Although Allais's and Machina's theories are not contradicted by the reflection effect, they do not predict it.

Fishburn's model is more like regret theory (although he does not mention any notion of regret) in that he also drops the transitivity axiom. However, his model is presented in terms of prospects rather than actions, and therefore does not accommodate the isolation effect. On the other hand, if we restrict ourselves to statistically independent prospects (and Fishburn does so – see his p. 9), then our theory and his basic axioms are compatible, and provide an interesting example of how an axiomatic treatment and a more introspective psychologically-based approach may complement each other.¹

However, having indicated that our theory provides certain predictions and explanations that the other theories mentioned do not, we should make it clear that we are not claiming that regret theory can explain *all* of the behavioural regularities revealed by experimental research into choice under uncertainty. So far we have focused on a number of patterns of behaviour observed by Kahneman and Tversky; but we have not dealt with every one of their observations, still less with the vast amount of evidence accumulated by other researchers.

Some of the experimental findings do not appear to be completely consistent. In relation to this paper, the most significant case concerns the reflection effect. Hershey and Schoemaker (1980*a*) and Payne *et al.* (1980) have published results that show this effect to be not nearly as strong or as general as Kahneman and Tversky's evidence suggests. However, this may not present any great difficulties for regret theory since, as we noted in Section III (*d*), the *general* prediction of the reflection effect requires $C(\cdot)$ to be linear. Instances in which the reflection effect is weak or absent may well be explicable if $C(\cdot)$ is assumed to be concave.

There are nevertheless certain observations that simply cannot be accounted for by regret theory in the form presented here. One example is the 'framing' effect discussed by Tversky and Kahneman (1981) and the very similar 'context' effect observed by Hershey and Schoemaker (1980*b*). In these cases exactly the

¹ At a late stage, we have received a copy of a Working Paper by David E. Bell (1981) which is of great interest. Quite independently he has developed a model which also explicitly incorporates a notion of regret, using multi-attribute utility theory along the lines suggested by Keeney and Raiffa (1976). We note that when both models are applied to the same phenomena – the original Allais paradox, simultaneous insuring and gambling, and the reflection effect – the conclusions are strikingly similar.

same choice problem – that is, exactly the same when formulated in terms of a matrix of state-contingent consequences – receives markedly different responses, depending on the way the choice is presented. Another example is the ‘translation’ effect observed by Payne *et al.* (1980). This effect occurs when an individual prefers one prospect to another, but reverses his preference when the same sum of money is deducted from every consequence of both prospects. The observed pattern of reversal is not predicted by regret theory. Finally, systematic violations of the sure-thing principle have been observed (cf. Moskowitz (1974); Slovic and Tversky (1974)). And although there is some evidence that individuals violate the sure-thing principle much less often than they violate some other axioms (Tversky and Kahneman (1981, footnote 15)), as it stands our theory does not explain that behaviour.

On the other hand, there is some additional evidence that gives further support to regret theory. A particular instance is the form of ‘preference reversal’ observed by Lindman (1971) and Lichtenstein and Slovic (1971, 1973) and subsequently confirmed, after rigorous testing, by Grether and Plott (1979). This preference reversal occurs when an individual, faced with a pairwise choice between gambles *A* and *B*, chooses *A*; but when asked to consider the two gambles separately, places a higher certainty equivalent value on *B*. We have shown elsewhere (Loomes and Sugden (1982)) that the most commonly observed reversal pattern is predicted by regret theory even in its restricted form.

Of course, we acknowledge that there is no simple theory that gives a unified explanation of all the experimental evidence, and regret theory is no exception in this respect. But we have tried to construct a theory that explains as much of the evidence as possible on the basis of very few assumptions. We do not believe that choiceless utility and regret are the only factors that influence behaviour under uncertainty, but just that these two factors seem to be particularly significant. Indeed, we have become increasingly convinced by evidence of framing, context and translation effects that the notion of reference points deserves further consideration, although we have not tried to deal with that issue in this paper.

In constructing our theory we have avoided any assumptions of misperceptions or miscalculations by individuals. We do not doubt that in reality misperceptions and miscalculations occur, and sometimes in systematic rather than random ways. Nonetheless, our inclination as economists is to explain as much human behaviour as we can in terms of assumptions about rational and undeceived individuals. Thus we believe that regret theory does more than predict certain systematic violations of conventional expected utility theory: it indicates that such behaviour is not, in any meaningful sense of the word, irrational.

In claiming this we are breaking the terms of a truce that many theorists (with the notable exception of Allais) have tacitly accepted. Proponents of expected utility theory often concede that their theory has serious limitations as a predictive device but insist that its axioms have strong normative appeal as principles of rational choice. Thus Morgenstern (1979, p. 180) argues for expected utility theory on the grounds that ‘if people deviate from the theory, an explanation of the theory and of their deviation will cause them to re-adjust their behaviour’. Similarly, Savage (1954, pp. 102–3) admits that when confronted with a pair of

choice problems rather like Problems 1 and 2, he behaved in accordance with the common consequences effect and in violation of his own axioms. But, he says, he was able to convince himself that this behaviour was mistaken (though even after realising his 'mistake' he continued to feel an 'intuitive attraction' to that behaviour). At the other side of the truce, proponents of alternative theories have often been willing to accept these claims. Kahneman and Tversky (1979, p. 277) maintain that the departures from expected utility theory that prospect theory describes 'must lead to normatively unacceptable consequences' which a decision-maker would, if he realised the error of his ways, wish to correct. Similarly, Machina (1982, p. 277) notes the 'normative appeal' of the axioms of expected utility theory before going on to propose a positive theory that dispenses with one of these axioms.

However, we shall challenge the idea that the conventional axioms constitute the only acceptable basis for rational choice under uncertainty. We shall argue that it is no less rational to act in accordance with regret theory, and that conventional expected utility theory therefore represents an unnecessarily restrictive notion of rationality.

Regret theory rests on two fundamental assumptions: first, that many people experience the sensations we call regret and rejoicing; and second, that in making decisions under uncertainty, they try to anticipate and take account of those sensations.

In relation to the first assumption, it seems to us that psychological experiences of regret and rejoicing cannot properly be described in terms of the concept of rationality: a choice may be rational or irrational, but an experience is just an experience. As far as the second assumption is concerned, if an individual does experience such feelings, we cannot see how he can be deemed irrational for consistently taking those feelings into account.

We do not claim that acting according to our theory is the *only* rational way to behave. Nor do we suggest that all individuals who act according to our theory must violate the conventional axioms. Some individuals may experience no regret or rejoicing at all, while some others may have linear $Q(\cdot)$ functions: in these special cases of our theory, we would predict that the individual's behaviour would conform with all the conventional axioms.

On the other hand, individuals with non-linear $Q(\cdot)$ functions of the kind described in this paper may consistently and knowingly violate the axioms of transitivity and equivalence without ever accepting, even after the most careful reflection, that they have made a mistake. So these axioms do not necessarily have the self-evident or overwhelming normative appeal that many theorists suppose. We shall now try to show why we do not accept the idea that the transitivity and equivalence axioms are necessary conditions for rational choice under uncertainty.

Underlying those two axioms is a common idea: that the value placed on any action A_i depends only on the interaction between, on the one hand, the probability-weighted consequences offered by A_i and, on the other hand, the individual's pattern of tastes, including his attitude to risk.

That is what is symbolised when, for any individual, an expected utility

number is assigned to an action, that expected utility number being quite independent of the range and nature of the available alternative actions. From this idea, that there is some value in 'having A_i ' which is quite independent of the value of 'having A_k ', and that if 'having A_i ' gives more value than 'having A_k ' then $A_i \succ A_k$, it follows that there must exist a complete and transitive preference ordering over all actions.

It also follows that the particular state pattern of consequences is of no special significance: if each action is evaluated independently, it does not matter how the consequence of that action under any state of the world compares with the consequence(s) of any other action(s) under the same state. Thus only the probability distribution of consequences matters, and all actions, simple or complex, which share the same probability distribution will be assigned the same expected utility number and must be regarded as equivalent for the purposes of choice decisions.

But if people experience regret and rejoicing, these arguments are illegitimate. In regret theory the proposition $A_i \succcurlyeq A_k$ cannot be read as 'having A_i is at least as preferred as having A_k '; it should rather be read as 'choosing A_i and simultaneously rejecting A_k is at least as preferred as choosing A_k and simultaneously rejecting A_i '. Thus the transitivity of the relation 'is at least as preferred as' (which we do not dispute) does not entail the transitivity of our relation \succcurlyeq ; and so non-transitive choices do not indicate any logical inconsistency on the part of the decision-maker.

The idea that non-transitive choices are irrational is sometimes argued as follows. Suppose (as in the example discussed in connection with Table 4 in Section IV) that there are three actions A_1, A_2, A_3 , such that $A_1 \succ A_2, A_2 \succ A_3$, and $A_3 \succ A_1$. Then, it is said, no choice can be made from the set $\{A_1, A_2, A_3\}$ without there being an inconsistency with one of the original preference statements: whichever action is chosen, another is preferred to it (cf. MacKay (1980, p. 90)). The principle that is being invoked here is Chernoff's axiom: if A_i is chosen from some set S , and if S' is a subset of S that contains A_i , then A_i must be chosen from S' . But we suggest the appeal of this axiom derives from the supposition that the value of choosing an action is independent of the nature and combination of the actions simultaneously rejected; and regret theory does not accept this supposition. Since $A_1 \succ A_2$ means only that choosing A_1 from the set $\{A_1, A_2\}$ is preferred to choosing A_2 from the set $\{A_1, A_2\}$ there is no implication that choosing A_1 from the set $\{A_1, A_2, A_3\}$ is preferred to choosing A_2 from the set $\{A_1, A_2, A_3\}$. A similar argument applies to the example discussed in connection with Table 5 in Section IV, where (despite the fact that the relation \succcurlyeq happens to be transitive) there is another violation of Chernoff's axiom.

A second common objection to non-transitivity runs like this. If someone prefers A_1 to A_2, A_2 to A_3 , and A_3 to A_1 , every one of the actions is less preferred than another; so might he not get locked into an endless chain of choice in which he can never settle on any one action? Worse, might not a skilful bookmaker capture all his wealth by confronting him with a suitably constructed sequence of pairwise choices? But these objections rest on a fallacy. To suppose that the individual can get locked into a cycle of choices, it is necessary to suppose that all

three actions are feasible. But if this is indeed the case, then propositions about pairwise choices – about how choices are made when there are only two feasible actions – are not relevant. The bookmaker can bankrupt his client only if he can successively persuade him to believe in each of a long chain of mutually inconsistent propositions about the feasible set.

Finally, there is no reason why the equivalence axiom should be regarded as a necessary condition for rational choice, even when the choice is between two simple actions with identical probability distributions of consequences. Consider

Table 6

Action	0.25	0.25	0.25	0.25
A_i	3	2	1	0
A_k	0	3	2	1

A_i and A_k in Table 6. If each action were evaluated independently, there would be no grounds for preferring ‘having A_i ’ to ‘having A_k ’, or vice versa. But in our model the decision is between ‘choosing A_i and simultaneously rejecting A_k ’ and ‘choosing A_k and simultaneously rejecting A_i ’. These two alternatives are associated with different probability mixes of regret and rejoicing. (In terms of our theory, to choose A_i and reject A_k is to incur a 0.25 probability of $R(+3)$ and a 0.75 probability of $R(-1)$, while to choose A_k and reject A_i is to incur a 0.25 probability of $R(-3)$ and a 0.75 probability of $R(+1)$.) So for an individual who experiences regret and rejoicing, the two courses of action cannot be regarded as identical. It would therefore not be unreasonable for such an individual to prefer one to the other.

VI. CONCLUSION

The evidence presented by Kahneman and Tversky and many others points to a number of cases where commonly observed patterns of choice violate conventional expected utility axioms. The fact that these violations are neither small-scale nor randomly distributed may indicate that there are some important factors affecting many people’s choices which have been overlooked or mis-specified by conventional theory.

We suggest that one significant factor is an individual’s capacity to anticipate feelings of regret and rejoicing. We therefore offer an alternative model which takes those feelings into consideration. This model yields a range of predictions consistent with the behaviour listed in Table 1 and provides an account of these and other choice phenomena which conventional theory has so far failed to explain.

That is the positive side of regret theory. But we believe that our approach also has strong normative implications. We have argued that our theory describes a form of behaviour which, although contravening the axioms of expected utility theory, is rational. Thus, while we do not suggest that behaving according to those

conventional axioms is irrational, we *do* suggest that those axioms constitute an excessively restrictive definition of rational behaviour.

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Date of receipt of final typescript: April 1982

Appendix: Inferring subjective probabilities and $C(\cdot)$ and $Q(\cdot)$ functions from choices

The following procedure will reveal, for any individual, which of two events has the higher subjective probability. Let S_1 and S_2 be any two non-intersecting and non-empty events (i.e. sets of states of the world). Let S_3 be the event that comprises all those states of the world not in S_1 or S_2 . Let x, y, z be any three consequences such that the person in question prefers x to y (under certainty). Consider the two actions $A_i = (x, y, z)$ and $A_k = (y, x, z)$, which are defined in relation to the events S_1, S_2, S_3 . It then follows from the separability principle (see Section III) that A_i is preferred to, indifferent to, or less preferred than A_k as the subjective probability of S_1 is greater than, equal to, or less than that of S_2 . This procedure is broadly similar to the one proposed by Savage (1954) for inferring subjective probabilities for individuals who behave according to his postulates.

The restricted form of our theory (see Section III) uses two functions for the analysis of modified utility: $C(\cdot)$ and $Q(\cdot)$. $C(\cdot)$ can be identified, up to a positive linear transformation, by confronting the individual with choices involving 50–50 gambles. Consider any two prospects of the form $X_i = (x_1, 1)$, $X_k = (x_2, 0.5; x_3, 0.5)$ where $x_3 > x_1 > x_2$, so that the corresponding choiceless utility indices are $c_3 > c_1 > c_2$. Then:

$$X_i \begin{matrix} \succ \\ \cong \\ \prec \end{matrix} X_k \text{ iff } 0.5 Q(c_1 - c_2) - 0.5 Q(c_3 - c_1) \begin{matrix} \cong \\ > \\ < \end{matrix} 0.$$

But since $Q(\cdot)$ is increasing, it follows that:

$$X_i \begin{matrix} \succ \\ \cong \\ \prec \end{matrix} X_k \text{ iff } 0.5 (c_1 - c_2) - 0.5 (c_3 - c_1) \begin{matrix} \cong \\ > \\ < \end{matrix} 0.$$

Thus in this case, the individual chooses *as though* maximising expected *choiceless* utility. So $C(\cdot)$ can be identified from experiments in much the same way as von Neumann–Morgenstern utility functions are identified.

If $C(\cdot)$ is known, and if a particular transformation has been chosen, it is possible to define consequences in terms of their choiceless utilities. Let x_1 and x_2 be consequences such that $c_1 = 0$ and $c_2 = -1$. Let x_3 be any consequence such that $c_3 = \xi$ where $\xi > 0$ and $\xi \neq 1$. Consider the two prospects $X_i = (x_1, 1)$ and $X_h = (x_2, p; x_3, 1 - p)$. Then:

$$X_i \begin{matrix} \succ \\ \cong \\ \prec \end{matrix} X_h \text{ iff } \frac{Q(\xi)}{Q(1)} \begin{matrix} \cong \\ > \\ < \end{matrix} \frac{p}{1-p}.$$

Thus if one can find a value of p such that the individual is indifferent between X_i and X_h it is possible to infer the value of $Q(\xi)/Q(1)$. So if $Q(1)$ is set equal to any arbitrary positive value, the value of $Q(\xi)$ can then be determined by experiment for all $\xi > 0$; hence the concavity, convexity or linearity of $Q(\cdot)$ over any interval can be established.

REFERENCES

- Allais, M. (1953). 'Le comportement de l'homme rationnel devant le risque; critique des postulats et axiomes de l'école Américaine.' *Econometrica*, vol. 21, pp. 503–46.
- and Hagen, O. (1979). *Expected Utility Hypothesis and the Allais Paradox*. Dordrecht: Reidel.
- Bell, D. E. (1981). 'Regret in decision-making under uncertainty.' *Harvard Business School Working Paper* 82–15.
- Fishburn, P. C. (1981). 'Nontransitive measurable utility.' *Bell Laboratories Economics Discussion Paper* 209.
- Friedman, M. and Savage, L. J. (1948). 'The utility analysis of choices involving risks.' *Journal of Political Economy*, vol. 56, pp. 279–304.
- Grether, D. M. and Plott, C. R. (1979). 'Economic theory of choice and the preference reversal phenomenon.' *American Economic Review*, vol. 69, pp. 623–38.
- Hershey, J. C. and Schoemaker, P. J. H. (1980a). 'Prospect theory's reflection hypothesis: a critical examination.' *Organizational Behavior and Human Performance*, vol. 25, pp. 395–418.
- and — (1980b). 'Risk taking and problem context in the domain of losses: an expected utility analysis.' *Journal of Risk and Insurance*, vol. 47, pp. 111–32.
- Hirschleifer, J. (1966). 'Investment decisions under uncertainty: applications of the state preference approach.' *Quarterly Journal of Economics*, vol. 80, pp. 252–77.
- Kahneman, D. and Tversky, A. (1979). 'Prospect Theory: an analysis of decision under risk.' *Econometrica*, vol. 47, pp. 263–91.
- Keeney, R. L. and Raiffa, H. (1976). *Decisions with multiple objectives*. New York: Wiley.
- Lichtenstein, S. and Slovic, P. (1971). 'Reversals of preference between bids and choices in gambling decisions.' *Journal of Experimental Psychology*, vol. 89, pp. 46–55.
- and — (1973). 'Response-induced reversals of preference in gambling: an extended replication in Las Vegas.' *Journal of Experimental Psychology*, vol. 101, pp. 16–20.
- Lindman, H. R. (1971). 'Inconsistent preferences among gambles.' *Journal of Experimental Psychology*, vol. 89, pp. 390–7.
- Loomes, G. and Sugden, R. (1982). 'A rationale for preference reversal.' *Newcastle Discussion Papers in Economics* No. 63.
- Machina, M. J. (1982). "'Expected utility" analysis without the independence axiom.' *Econometrica*, vol. 50, pp. 277–323.
- MacKay, A. F. (1980). *Arrow's Theorem: the Paradox of Social Choice*. New Haven, Conn.: Yale University Press.
- Markowitz, H. M. (1952). 'The utility of wealth.' *Journal of Political Economy*, vol. 60, pp. 151–8.
- Morgenstern, O. (1979). 'Some reflections on utility.' In *Expected Utility Hypotheses and the Allais Paradox*. (Ed. M. Allais and O. Hagen). Dordrecht: Reidel.
- Moskowitz, H. (1974). 'Effects of problem presentation and feedback on rational behavior in Allais and Morlat type problems.' *Decision Sciences*, vol. 5, pp. 225–42.
- Payne, J. W., Laughunn, D. J. and Crum, R. L. (1980). 'Translation of gambles and aspiration level effects in risky choice behavior.' *Management Science*, vol. 26, pp. 1039–60.
- Savage, L. J. (1951). 'The theory of statistical decision.' *Journal of the American Statistical Association*, vol. 46, pp. 55–67.
- (1954). *The Foundations of Statistics*. New York: Wiley.
- Schoemaker, P. J. H. (1980). *Experiments on Decisions under Risk: the Expected Utility Hypothesis*. Boston: Martinus Nijhoff.
- (1982). 'The expected utility model: its variants, purposes, evidence and limitations.' *Journal of Economic Literature*, forthcoming.
- Slovic, P. and Tversky, A. (1974). 'Who accepts Savage's axiom?' *Behavioral Science*, vol. 19, pp. 368–73.
- Tversky, A. and Kahneman, D. (1981). 'The framing of decisions and the psychology of choice.' *Science*, vol. 211, pp. 453–8.
- von Neumann, J. and Morgenstern, O. (1947). *Theory of Games and Economic Behaviour*. (2nd edition). Princeton: Princeton University Press.